Optimization-based Control and Estimation

PRECEDE 2017

Prof. Matthias Preindl

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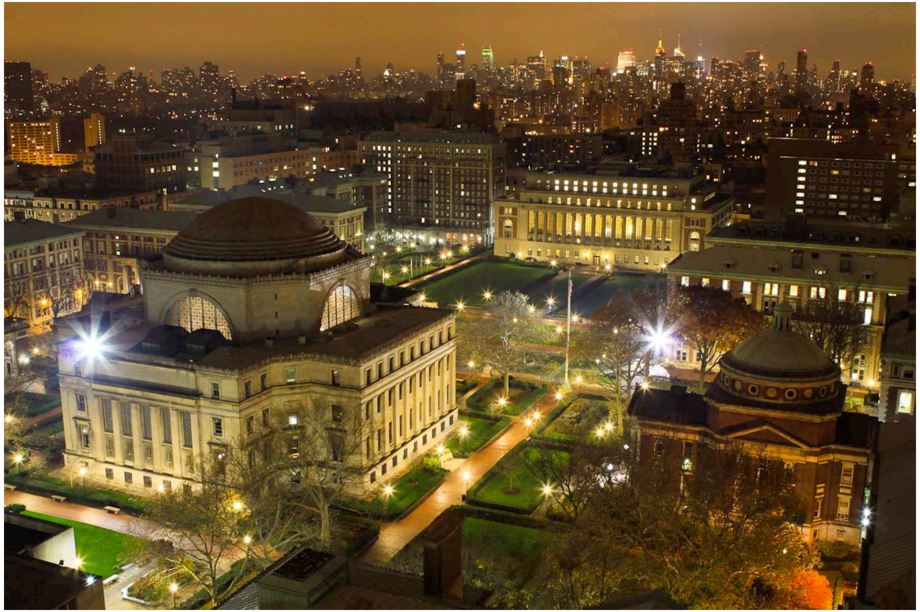
Columbia University in the City of New York

Ivy League Research University
Main Campus located in Manhattan, Morningside Heights

School of Engineering and Applied Science
~ 140 faculty members
~ 2,000 graduate students
~ 1,500 undergraduate students

Department of Electrical Engineering
- 36 faculty members
- Only one working in power



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Motor Drives and Power Electronics Laboratory (MPLab)

- Founded 2016
- Located on Columbia main campus

People

- Laboratory members
    4 PhD, 1 MSc students
- Co-supervision
    3 PhD students
    1 post-doc, 1 research associate
- Project-based members

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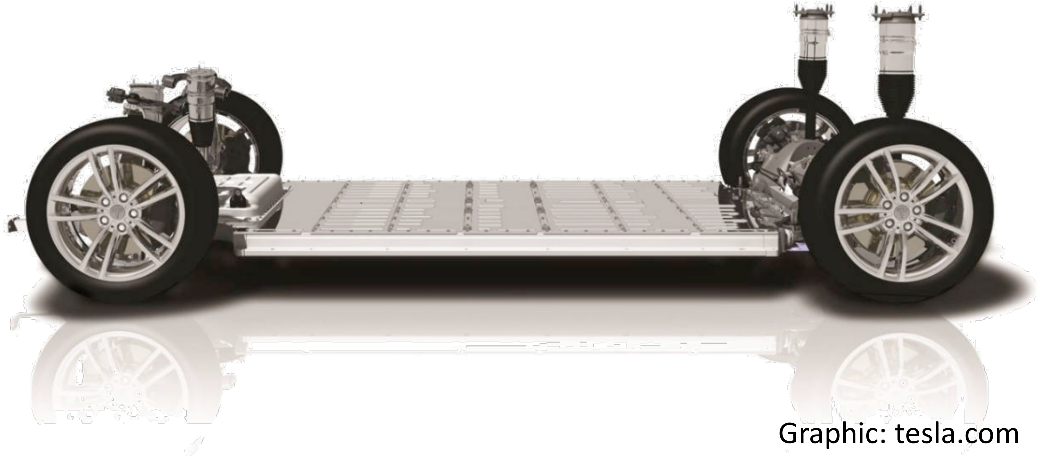
Research

- **Advanced control**
    - **Optimal control**
    - **Nonlinear control**
    - **Observers**

- Power electronics
    - Wide-bandgap
    - High-frequency (MHz)

Applications

- Electric Vehicle Drivetrains
    - Traction drive systems
    - Energy storage systems
    - Power converters



Graphic: tesla.com

Optimization-based Control and Estimation

Introduction

Introduction

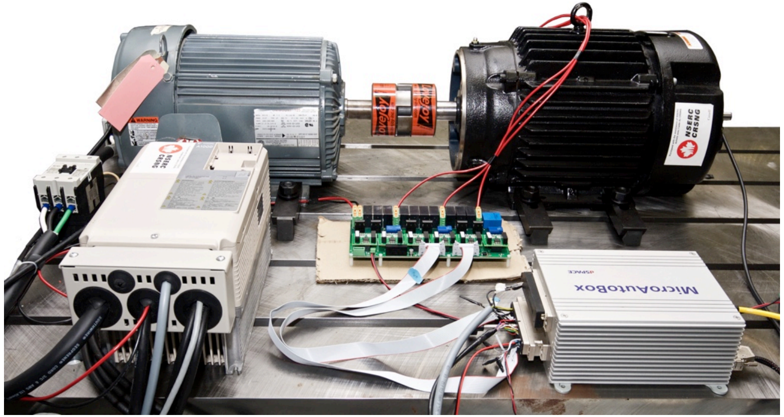
MPC Benefits

- Nonlinear Systems
- Multi-input multi-output
- Constraint handling

MPC Challenges

- “Classical” MPC stability theorem requires
  specific cost function, prediction horizon, and terminal constraint
- Convexity and computation efficiency
- Model accuracy

Observations



Virtual-flux model $(\lambda_{dq}, \lambda_{\alpha\beta})$

- Any multiphase AFE and sinusoidal machines: IPMSM, SPMSM, RSM, IM
- No parameters in dynamic model

MPC schemes

- FCS preferable at low f_s → needs to detect ripple
- CCS preferable at high f_s → ripple is handed off to modulator

Tendency to use unconventional cost functions

- Incompatibility with MPC theory: stability and robustness

Summary

- Single block is not be best for everything
- Stability and robustness concepts for any cost function
- MPC concepts are applicable to estimation
- MPC enables new power electronic topologies

Model Predictive Torque Control using Virtual Fluxes

Concept

Virtual flux λ space

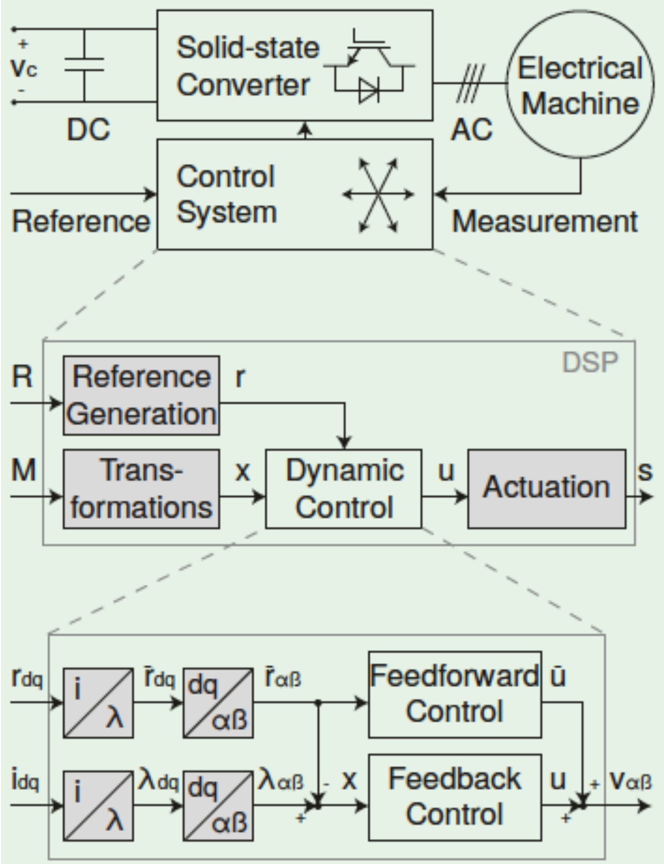
- Compute λ with static (nonlinear) map
- Linear dynamic model without parameters
- Prediction is parameter **independent**

Setpoint calculation

- Separate dynamic control from choosing setponts
- Nonlinear map: torque → current (or flux)

Regulation problem: $x \rightarrow 0$

- Tracking → regulation problem
- Combine feedback and feedworward control



Model Predictive Torque Control using Virtual Fluxes

Constrained MTPA

M. Preindl, S. Bolognani „Optimal State Ref. Computation with Constrained MTPA Criterion for PM Drives,“ TPEL, 2015

Model

Stator dynamics

$$\dot{\lambda}_{dq}(t) = -\omega J \lambda_{dq}(t) + \bar{v}_{dq}(t)$$

Current-flux map

$$\lambda_{dq}(t) = l \circ i_{dq}(t) \approx L i_{dq}(t) + \psi_{dq}$$

Torque equation

$$T = \frac{3}{2} p\, i'_{dq} J \lambda_{dq} = \frac{3}{2} p \left(\psi + (L_d - L_q) i_d\right) i_q$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}; \quad L = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix}; \quad \psi_{dq} = \begin{bmatrix} \psi \\ 0 \end{bmatrix};$$

Constraints

Current constraint

$$i_{dq} \in \mathcal{I} \stackrel{\text{def}}{=} \{ i_{dq} \in \mathbb{R}^2 \mid \|i_{dq}\| \leqslant I_r \}$$

Voltage constraint

$$\bar{v}_{dq} \in \bar{\mathcal{V}} \stackrel{\text{def}}{=} \left\{ \bar{v}_{dq} \in \mathbb{R}^2 \mid \|\bar{v}_{dq}\| \leqslant \bar{v}_r \stackrel{\text{def}}{=} \rho_v v_r \right\}$$

Flux constraint

$$\lambda_{dq} \in \Lambda \stackrel{\text{def}}{=} \{ \lambda_{dq} \in \mathbb{R}^2 \mid |\omega| \|\lambda_{dq}\| \leqslant \bar{v}_r \}$$

Speed constraint

$$\frac{|\omega|}{\bar{v}_r} \leqslant \chi_m \stackrel{\text{def}}{=} \begin{cases} \frac{1}{|\psi - L_d I_r|} & \text{if } \frac{\psi}{L_d} > I_r, \\ \infty & \text{otherwise.} \end{cases}$$

Constrained MTPA

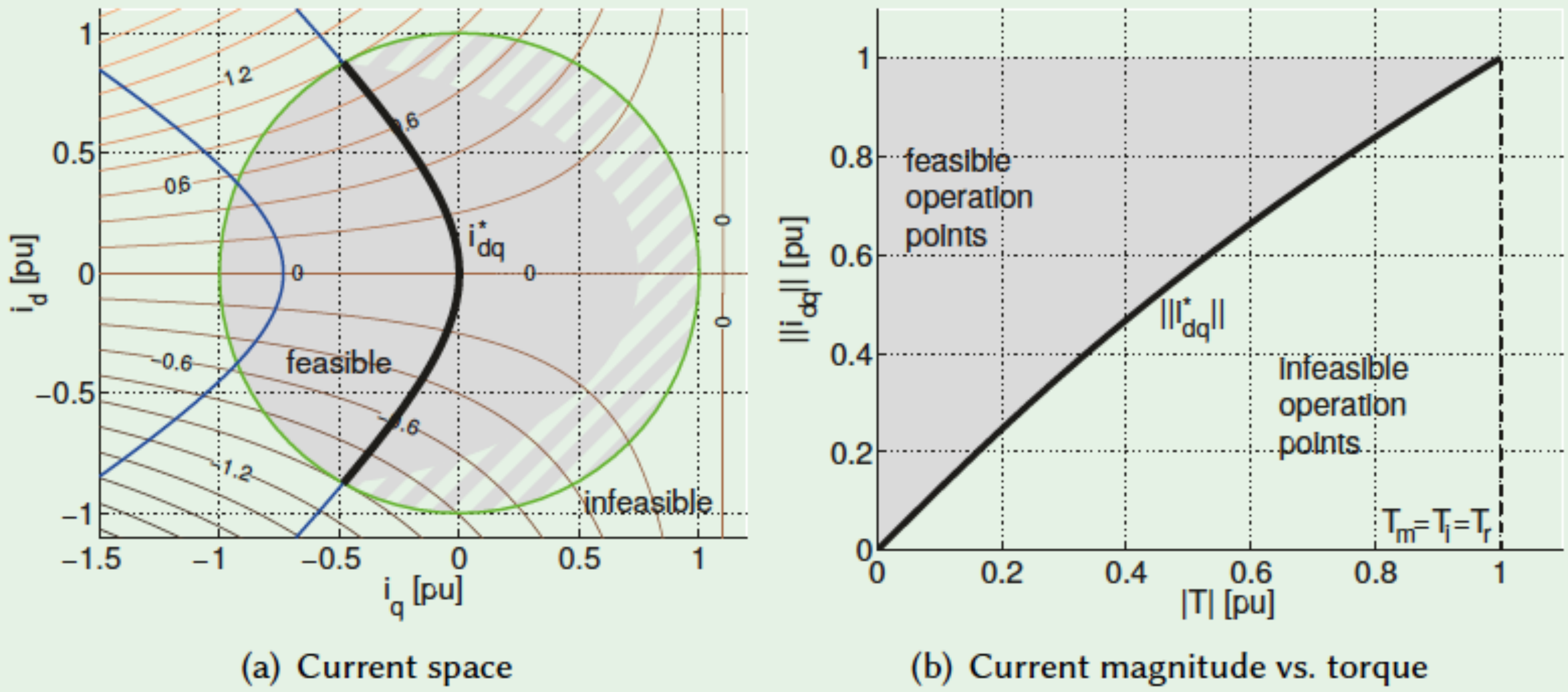
Definition: optimal states

Let $i^\star_{dq}, \lambda^\star_{dq}$ be optimal if they produce T according to

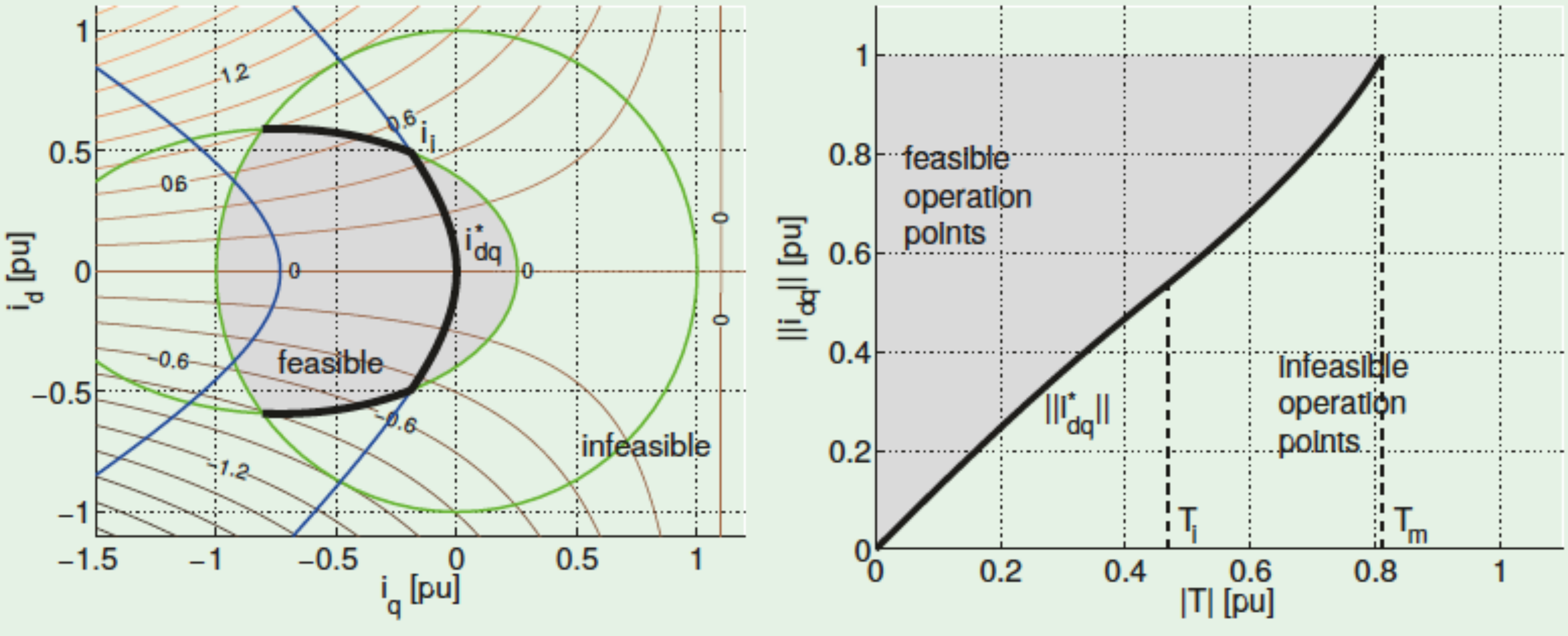
$$\begin{aligned} \underset{i_{dq},\lambda_{dq}}{\text{minimize}} \quad & \|i_{dq}\| \\ \text{subject to} \quad & \|i_{dq}\| \leqslant I_r; \\ & |\omega|\|\lambda_{dq}\| \leqslant \bar{v}_r; \\ & \lambda_{dq} = Li_{dq} + \psi_{dq}; \\ & 3/2p\, i'_{dq} J\lambda_{dq} = T; \\ & |T| \leqslant T_m(\omega, \bar{v}_r) \end{aligned}$$

Nonconvex (NP-hard in general), possibly infeasible → split into subproblems

Constrained MTPA – Example: Base Mode

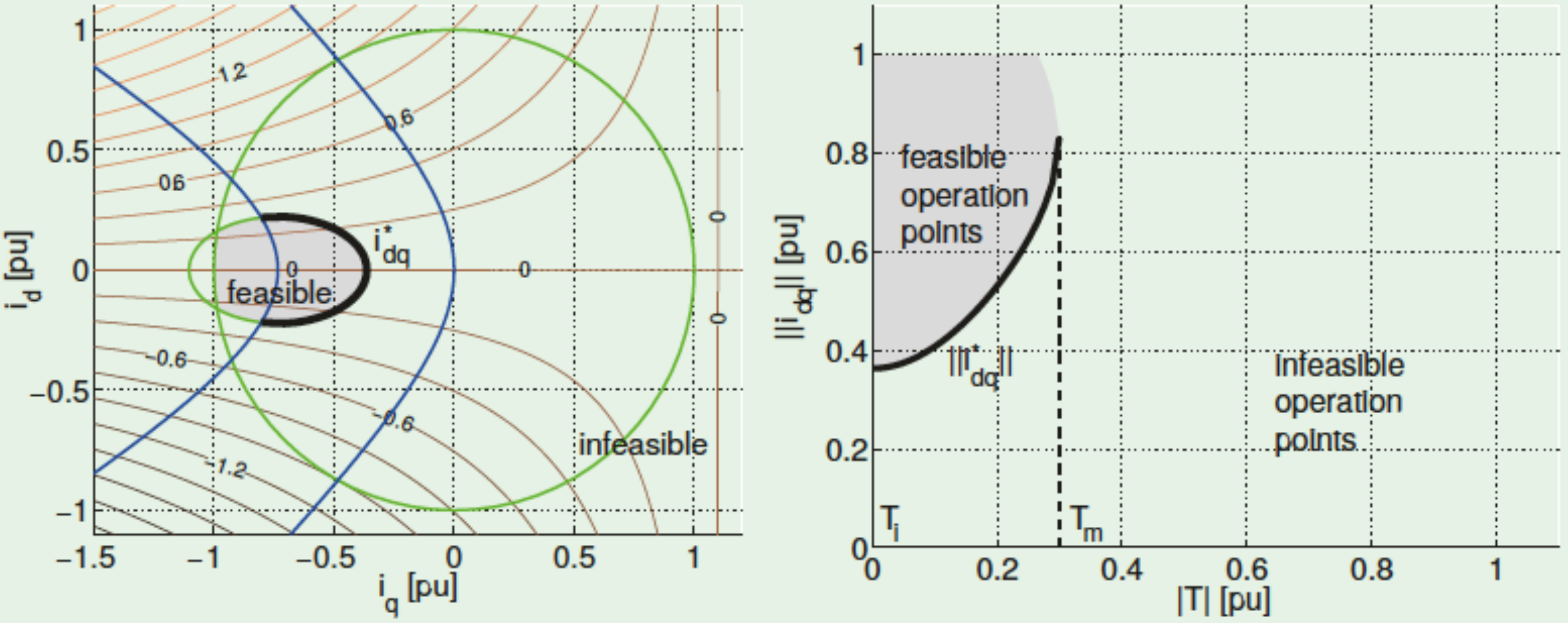


Constrained MTPA – Example: Constant Power Mode



(a) Current space (b) Current magnitude vs. torque

Constrained MTPA – Example: Reduced Power Mode



(a) Current space

(b) Current magnitude vs. torque

Constrained MTPA – T_m and T_i definitions

Definition: Maximum Torque

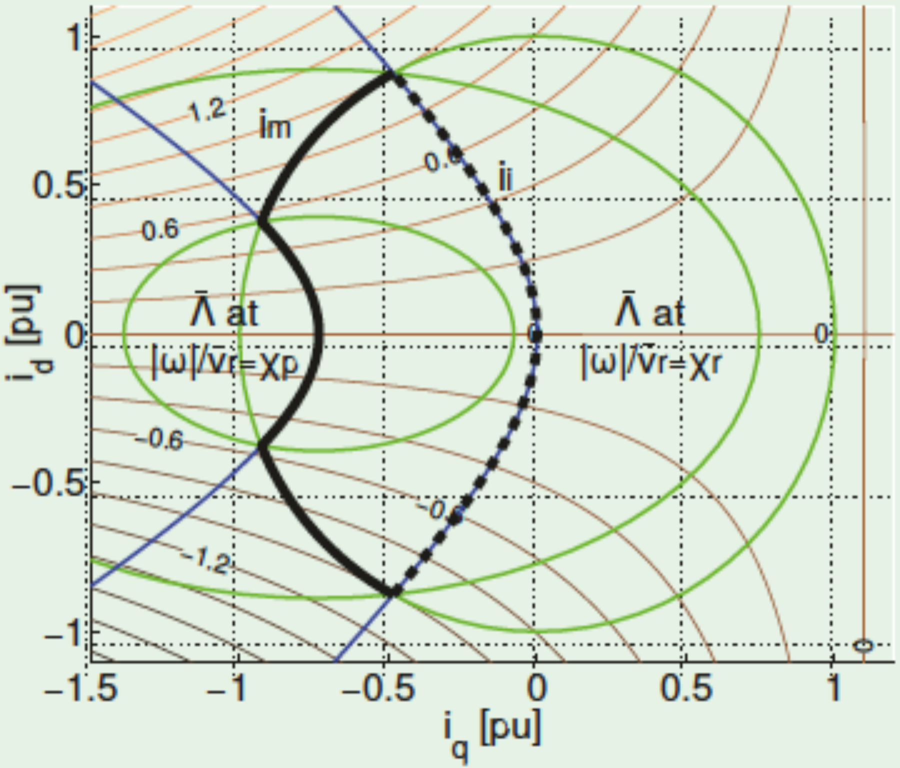
$$T_m \overset{\text{def}}{=} \frac{3}{2}p \max_{i_{dq},\lambda_{dq}} \; i_{dq}' J \lambda_{dq}$$
$$\text{subject to} \quad \|i_{dq}\| \leqslant I_r;$$
$$|\omega| \|\lambda_{dq}\| \leqslant \bar{v}_r;$$
$$\lambda_{dq} = L i_{dq} + \psi_{dq}$$

Definition: Intersection Torque

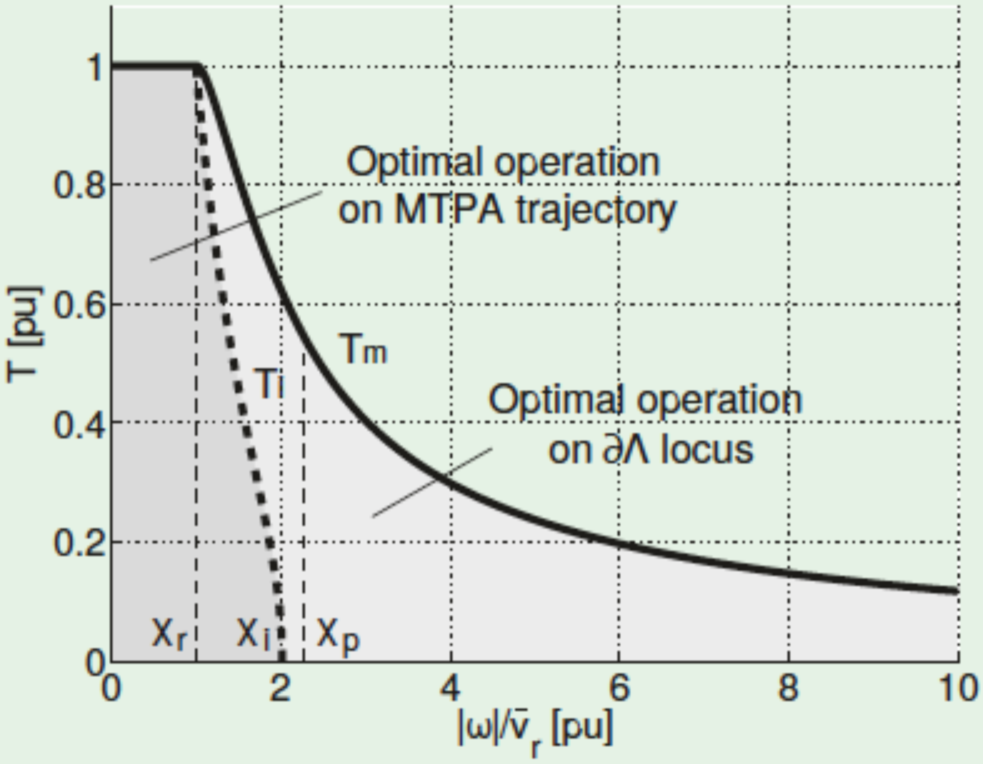
$$T_i \overset{\text{def}}{=} \frac{3}{2}p \max_{i_{dq},\lambda_{dq}} \; i_{dq}' J \lambda_{dq}$$
$$\text{subject to} \quad \|i_{dq}\| \leqslant I_r;$$
$$|\omega| \|\lambda_{dq}\| \leqslant \bar{v}_r;$$
$$\lambda_{dq} = L i_{dq} + \psi_{dq};$$
$$i_{dq} \in \text{MTPA}$$

- Problems are feasible;
- Still non-convex but can be solved efficiently due to low dimension

Constrained MTPA – T_m and T_i with infinite max. speed

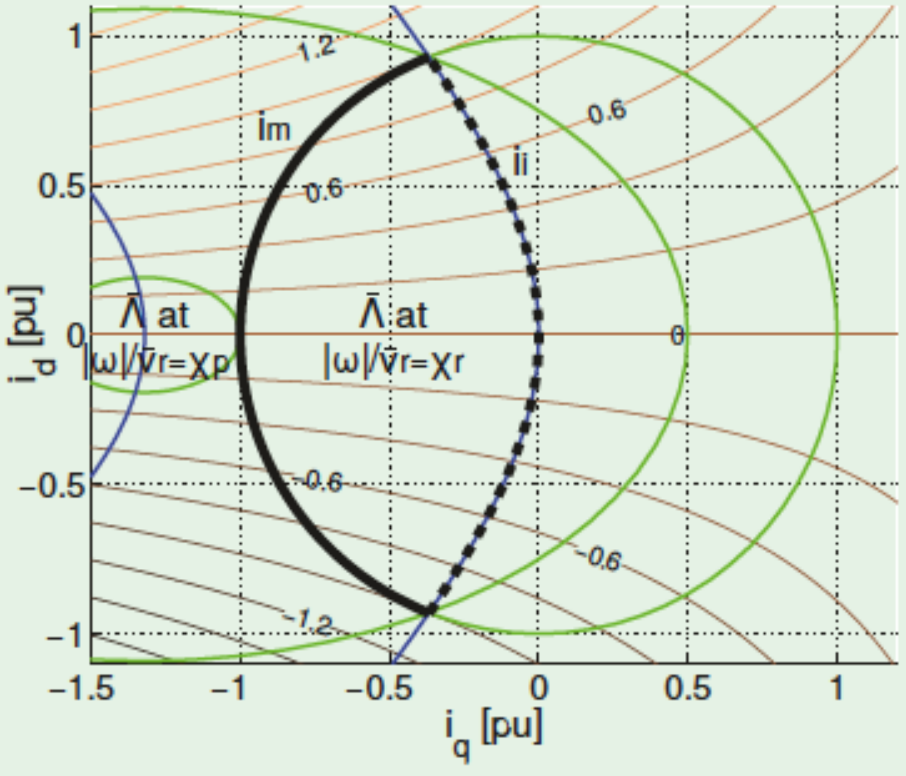


(a) Current space

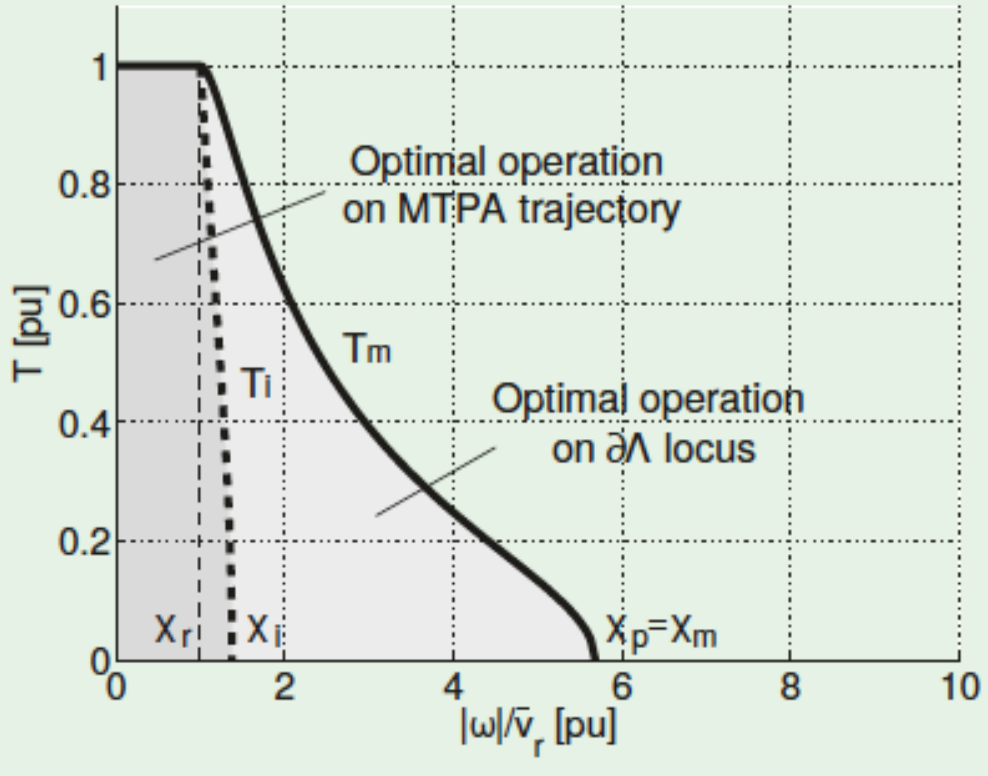


(b) Torque vs. normalized speed

Constrained MTPA – T_m and T_i with finite max. speed



(a) Current space



(b) Torque vs. normalized speed

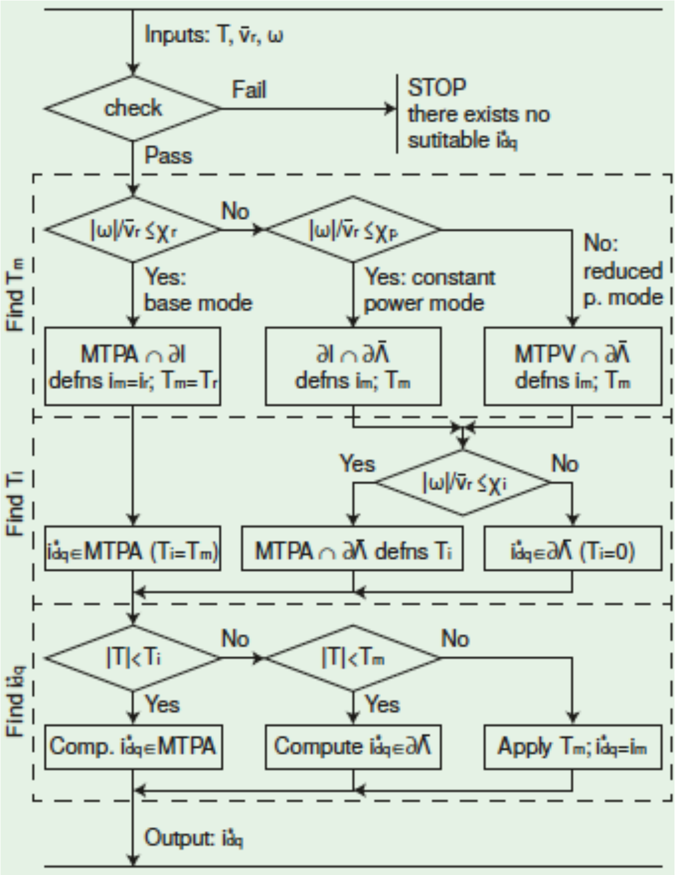
Constrained MTPA – Optimal Reference Computation

Find maximum torque T_m

Find intersection torque T_i

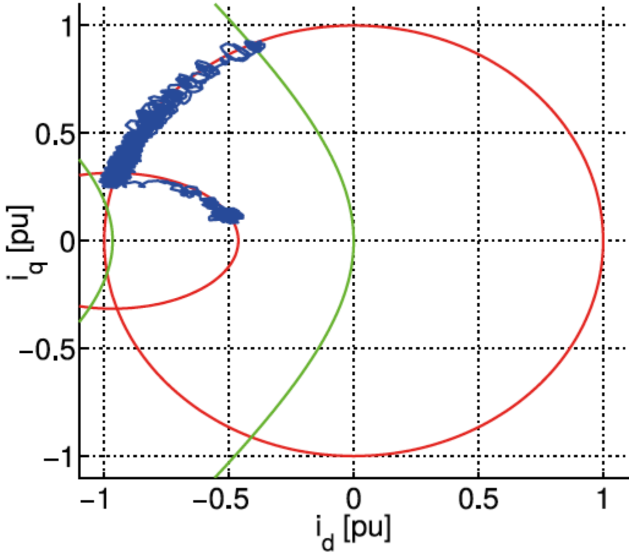
Find optimal states

- Locate trajectory (MTPA, $\partial\Lambda$) with T_m, T_i
- Compute $i_{dq,ref}$ (or $\lambda_{dq,ref}$)

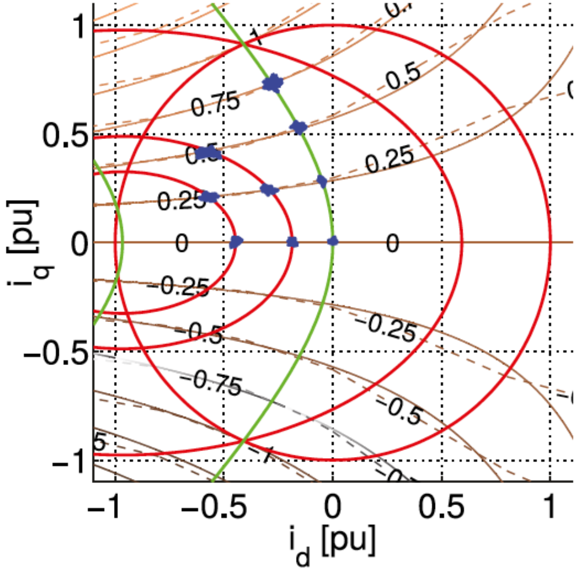


Constrained MTPA – Example: Low and High Speed Operation

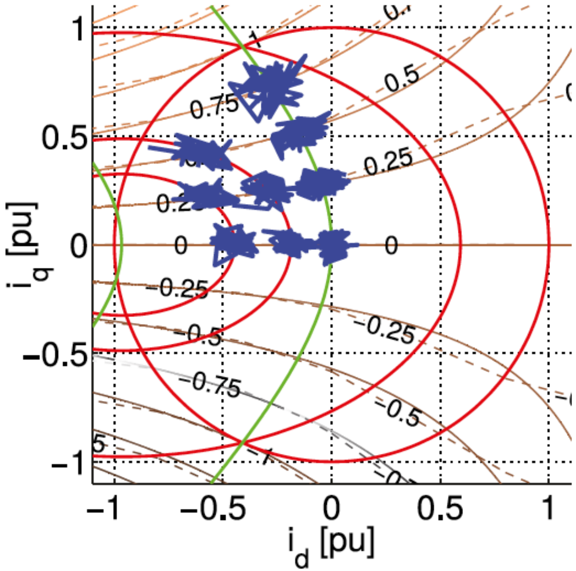
Find maximum torque



Transient operation (speed step)
standstill to field-weakening



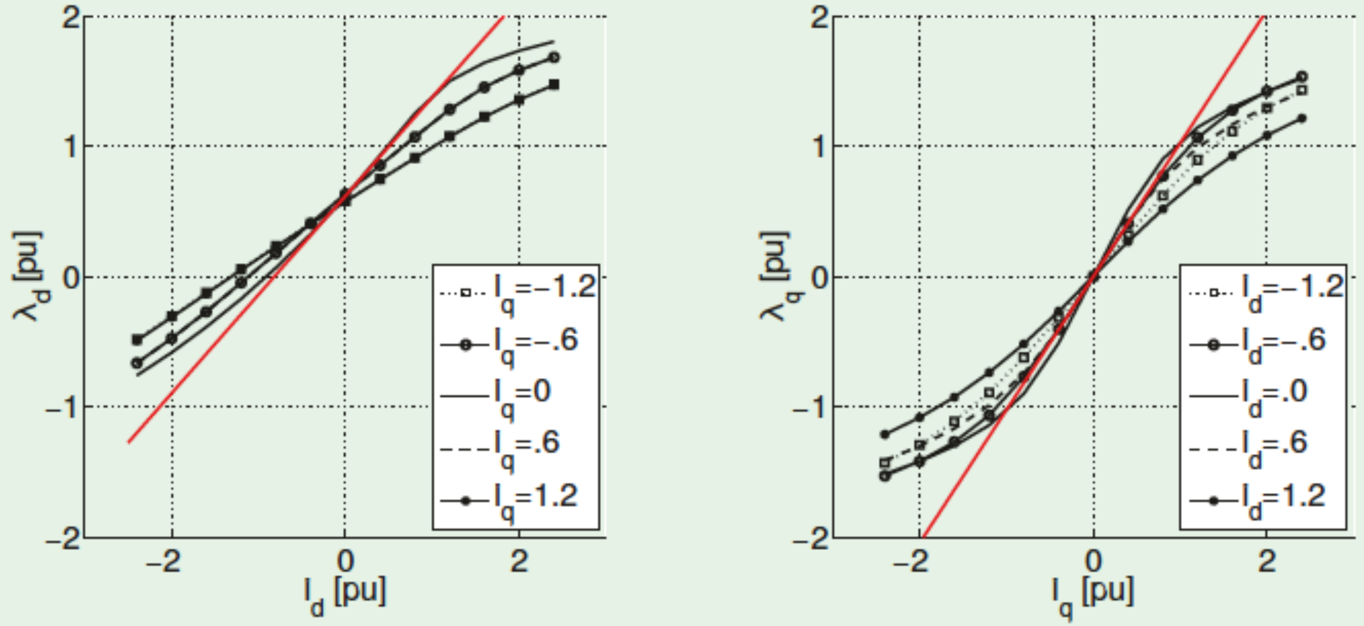
CCS-MPC
operation points



FCS-MPC
operation points

Constrained MTPA – Model Accuracy

Rated **L** and ψ_{dq} are typically suboptimal



(a) d-axis (b) q-axis

Constrained MTPA – Model Refinement

Approach

- Optimize model **locally** (area enclosed by MTPA, ∂Λ, and ∂I)
- Using well known operating points (rated operation point, short circ. current)

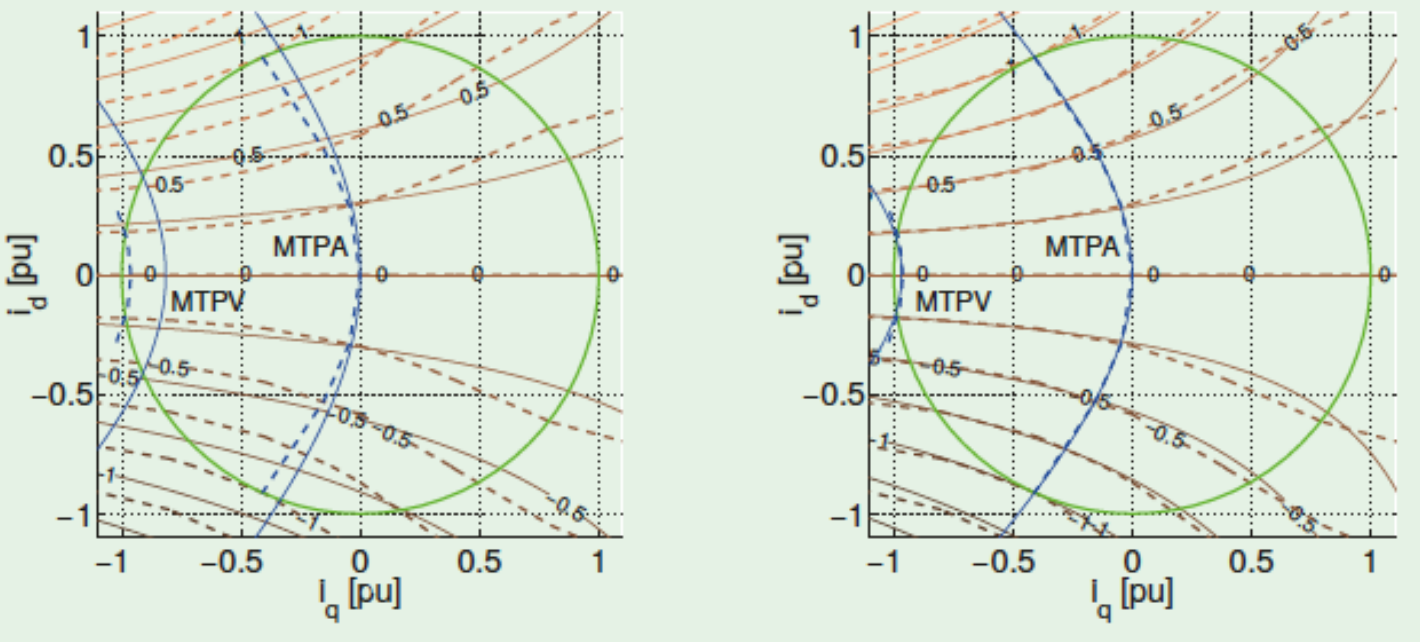
Parameters Mρ=K

$$\begin{bmatrix} \frac{3}{2} i_{qr} & \frac{3}{2} i_{dr} i_{qr} & -\frac{3}{2} i_{dr} i_{qr} & -T_r \\ i_d & i_{dr}^2 - i_{qr}^2 & i_{qr}^2 - i_{dr}^2 & \\ 1 & i_{dr} & & \\ & & i_{qr} & \\ 1 & i_{dc} & & \end{bmatrix} \begin{bmatrix} \psi \\ L_d \\ L_q \\ 1/p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \lambda_{dr} \\ \lambda_{qr} \\ 0 \end{bmatrix}$$

- Least square solution: ρ=**M**⁺K → not a general model, e.g. $p \notin \mathbb{N}$

Constrained MTPA – Example: Model Refinement

Model (solid) and measured (dashed) characteristics



(a) Rated model (b) Optimized model

Model Predictive Torque Control using Virtual Fluxes

Transient Behavior

M. Preindl „Robust Control Invariant Sets and Lyapunov-based MPC for IPM Sync. Motor Drives," TIE, 2016

Transient Behavior – Dynamic model

Stator **dynamic equation**

$$\bar{\lambda}_{\alpha\beta}^{+} = \bar{\lambda}_{\alpha\beta} + \bar{v}_{\alpha\beta}$$

Where $\bar{\lambda}_{\alpha\beta} = \Lambda_r^{-1}\lambda_{\alpha\beta}$ and $\Lambda_r = T_s v_c$

Transform tracking into **regulation problem**

$$x^{+} = x + u$$

with control error $x = \bar{\lambda}_{\alpha\beta} - \bar{r}_{\alpha\beta}$ and input $u = \bar{v}_{\alpha\beta} - \bar{u}$

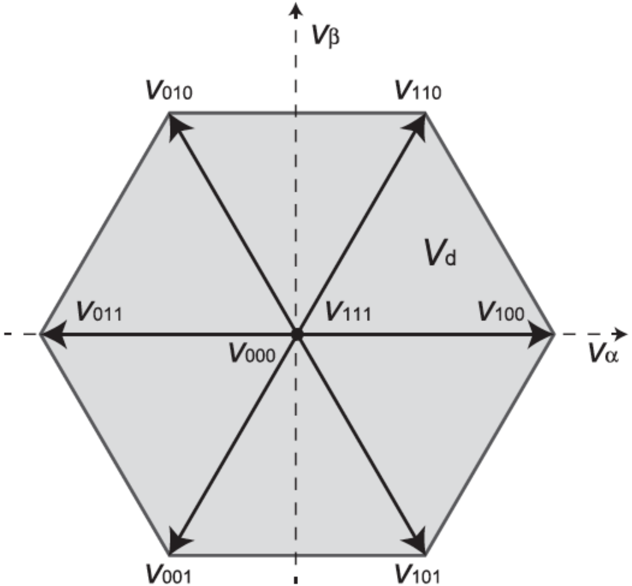
The terminal voltage $\bar{v}_{\alpha\beta}$

- Feedback controller u : MPC
- Feedforward controller $\bar{u}$: adjustment for rotation

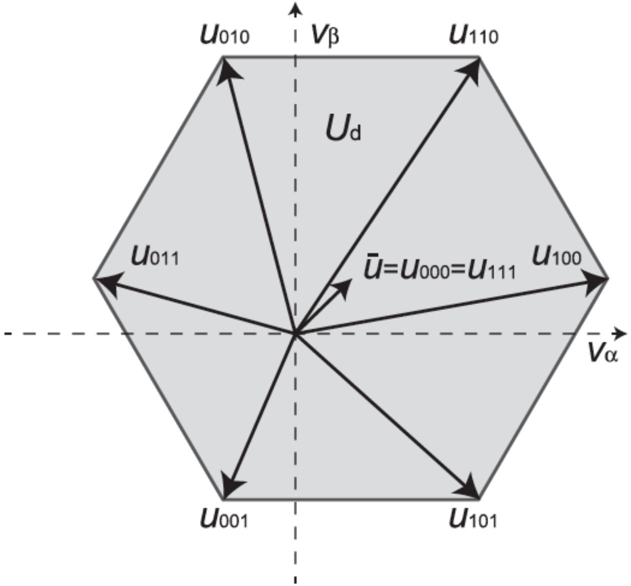
$$\bar{u} \approx -(\mathbf{I} - \mathbf{T}_{dq}^{-1}(T_s\omega))\bar{r}_{\alpha\beta}$$

Transient Behavior – Terminal Voltage and Input constraints

CCS and FCS input constraints (Note: $\mathcal{U}_s \stackrel{\text{def}}{=} \mathcal{V}_s - \bar{u}$ does not contain origin)

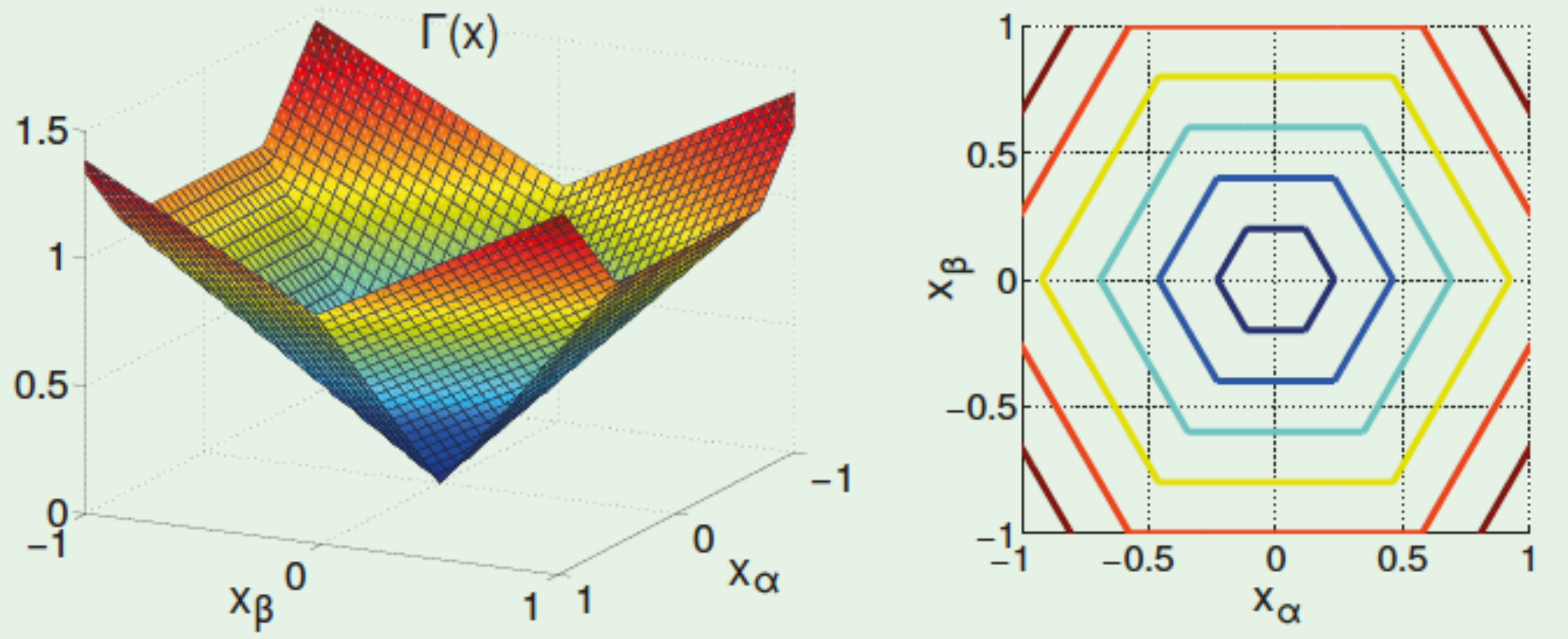


(a) Set $\mathcal{V}_s$ and $\mathcal{V}_d = \text{hull}\,\mathcal{V}_s$



(b) Set $\mathcal{U}_s$ and $\mathcal{U}_d = \text{hull}\,\mathcal{U}_s$

Transient Behavior – FCS Candidate Lyapunov Function



(a) $\Gamma(x) \overset{\text{def}}{=} \min_{x \in \Omega} X_l$ (b) $\Omega \overset{\text{def}}{=} \{u \in \mathbb{R}^2 \mid Hx \leqslant X_l\}$

Transient Behavior – Preset and Robust Control Invariance

Preset applied to sublevel set Ω_γ

The preset $O(\Omega_\gamma)$ is the set of all states $x \in \mathbb{R}^2$
that can be driven to the Ω_γ by an admissible control input $u \in U$

$$O_B(\Omega_\gamma) = \{x \in \mathbb{R}^2 \mid \exists u \in U : \mathrm{x} + \mathrm{u} \in \Omega_\gamma\}$$

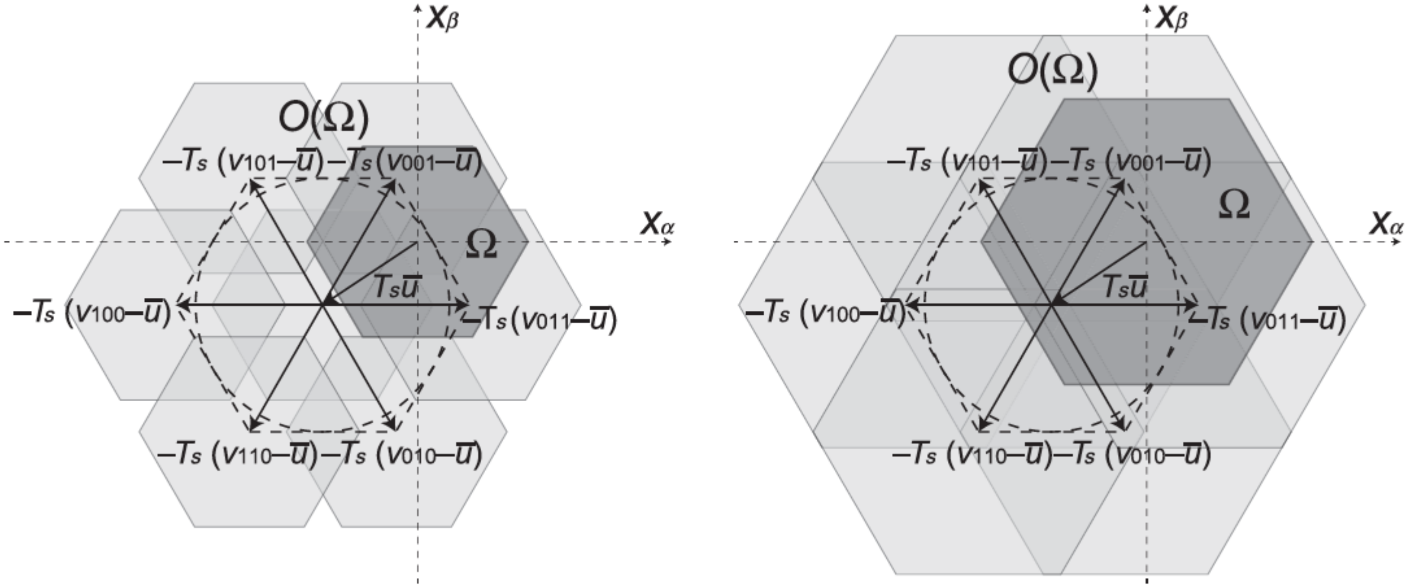
Robust control invariance

The set Ω_γ is said to be robust control invariant iff $\Omega_\gamma \subseteq O(\Omega_\gamma) - B$,
where B is an arbitrarily small ball with radius b

Iff Ω_γ is robust control invariant $\exists u \in U$ s.t. $\Gamma(x^+) - \Gamma(x) < -\mathrm{b}$.

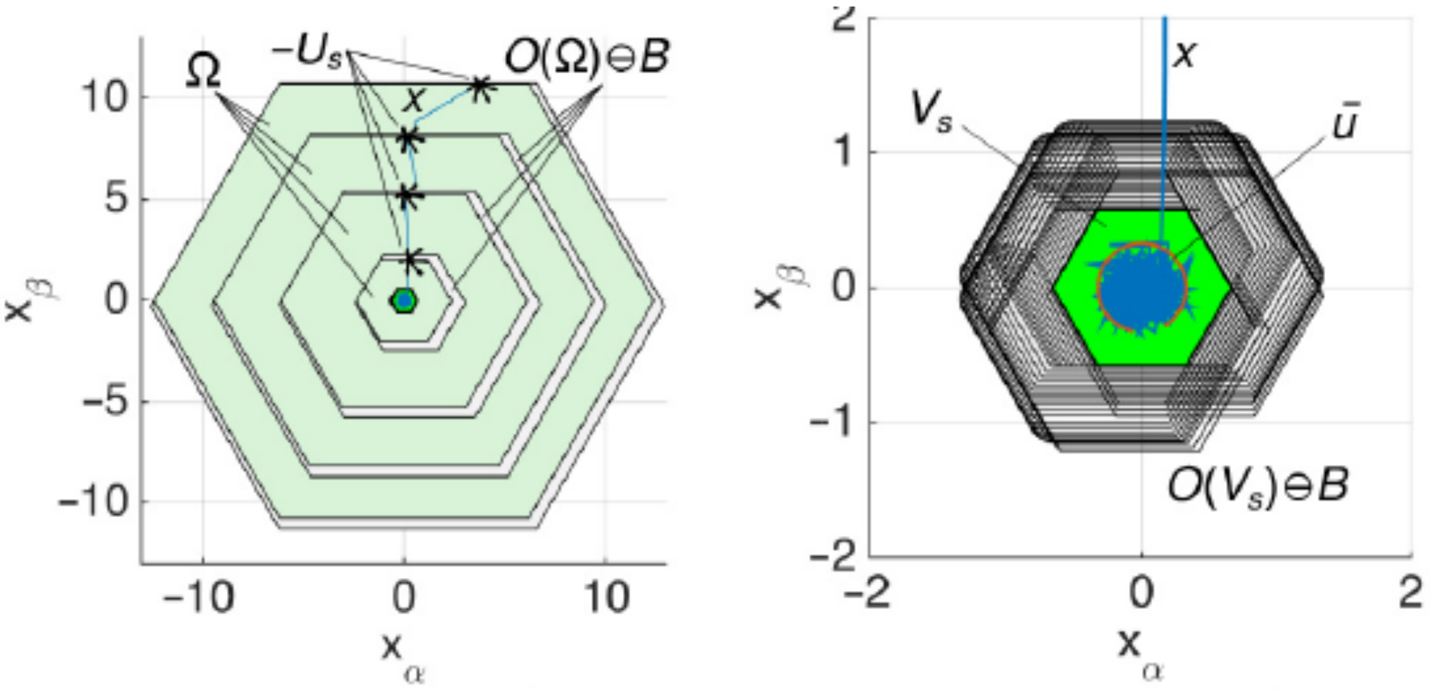
Transient Behavior – Control Invariance

Using FCS, the sublevel Ω_γ is (robust) control invariant if large enough



(a) $\Omega(\gamma)$ with $\gamma < \frac{1}{\sqrt{3}}$ (b) $\Omega(\gamma)$ with $\gamma \geq \frac{1}{\sqrt{3}}$

Transient Behavior – Example: presets and control invariance



(c) Transients ($x \notin \mathcal{V}_d$): $\mathcal{O}(\Omega(\gamma)) \ominus \mathcal{B}$ contains $\Omega(\gamma)$

(d) Steady state ($x \in \mathcal{V}_d$): $\mathcal{O}(\mathcal{V}_d) \ominus \mathcal{B}$ is rotating and contains $\mathcal{V}_d$

Transient Behavior – FCS Stabilizability: Theorem and Corollary

Theorem: global and robust **set** stabilizability

Let $\overline{u} \in V_d - B$, then $\exists u \in U_s$ s.t.

$$\Gamma(x^+) - \max\left(\Gamma(x), \frac{1}{\sqrt{3}} + b\right) < -b$$

The Lyapunov function can be decreased every time step by $-b$ until the level $\frac{1}{\sqrt{3}}$

Corollary: set convergence

There exists a sequence $u_0, u_1, \dots, u_k, \dots \in U_s$ s.t.

$$\lim_{k \to \infty} x_k \in V_\mathrm{d}$$

The control error converges to V_d

Transient Behavior – CCS Stabilizability: Theorem and Corollary

The CCS system inherits the FCS properties (without lower bound)

Theorem: global and robust stabilizability

Let $\overline{u} \in V_d - B$, then $\exists u \in U_\mathrm{d}$ s.t.

$$\Gamma(x^+) - \max(\Gamma(x), b) < -b$$

The Lyapunov function can be decreased every time step by $-b$ until the level $\frac{1}{\sqrt{3}}$

Corollary: convergence to origin

There exists a sequence $u_0, u_1, \dots, u_k, \dots \in U_\mathrm{s}$ s.t.

$$\lim_{k \to \infty} x_k \in 0$$

The control error converges to origin

Transient Behavior – Robust Lyapunov-based MPC

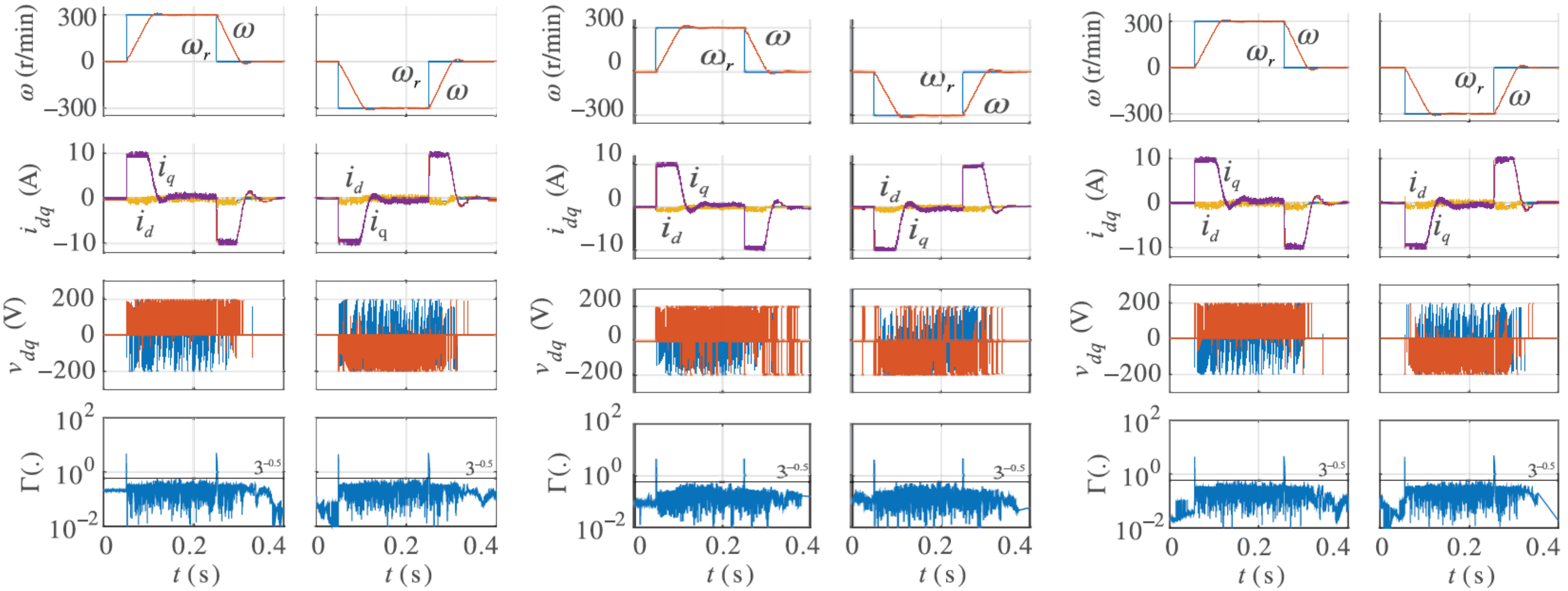
Constraint Finite Time Optimal Control (CFTOC)

- Enforce stability with contraction constraint
  CCS: any norm; FCS: specific candidate CLF
- Holds for any cost function
- Simplification possible for horizon N=1

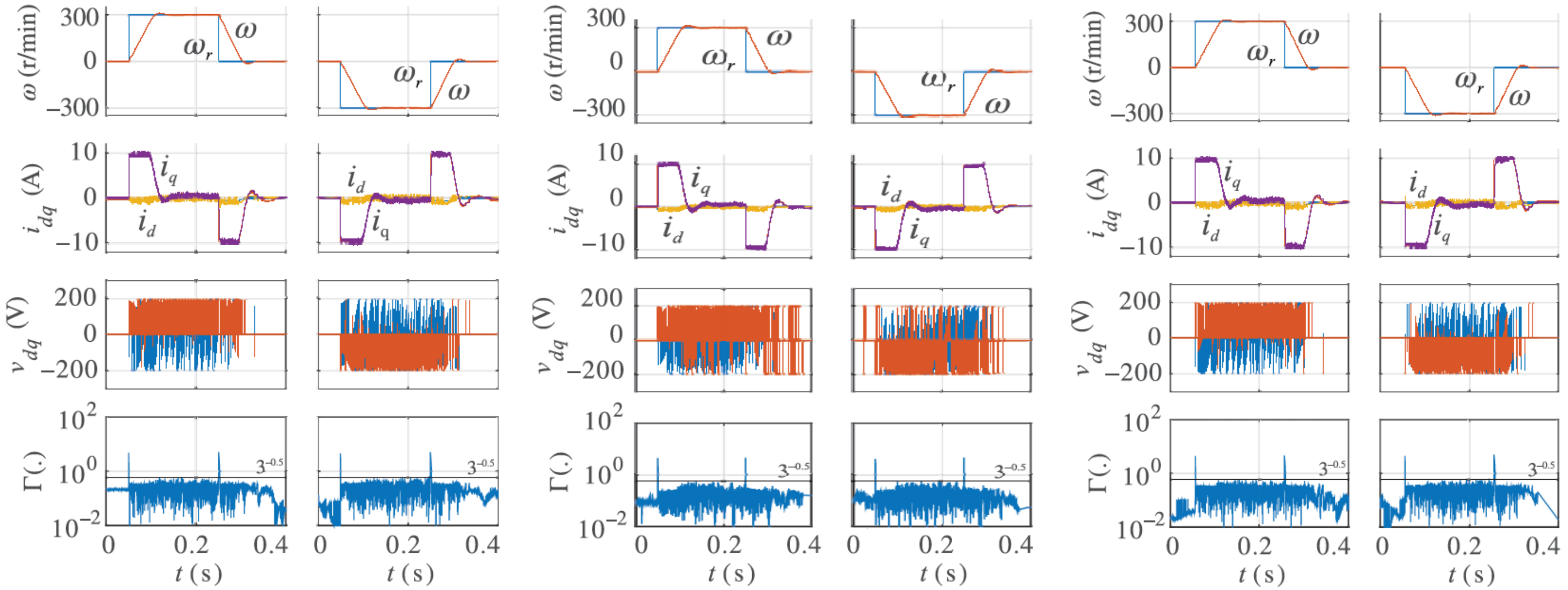
$$\begin{aligned}
&\underset{u_0,\ldots,u_{N-1}}{\text{minimize}}\; J(\cdot) \\
&\text{subject to}\; x_{j+1} = x_j + u_j \\
&\qquad u_j \in \mathcal{U}_j \stackrel{\text{def}}{=} \mathcal{V} - \bar{u}_j \\
&\qquad \Gamma(x_0 + u_0) - \max\left(\Gamma(x_0), \bar{\gamma} + b\right) \le -b
\end{aligned}$$

$\bar{\gamma} = \frac{1}{\sqrt{3}}$ for FCS; $\bar{\gamma} = 0$ for CCS

Transient Behavior – Example: Parameter Robustness



Transient Behavior – Example: Parameter Robustness



Smaller **L** by factor 100

Rated parameters

Larger **L** by factor 100

MPC for PM Synchronous Motor Drives

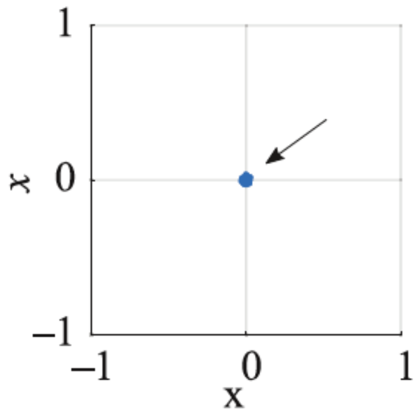
Steady-State Behavior

X. Yong, M. Preindl „Smallest Control Invariant Set and Error Boundaries of FCS-MPC for PMSM ,“ APEC, 2017

Steady-State Behavior – Observations

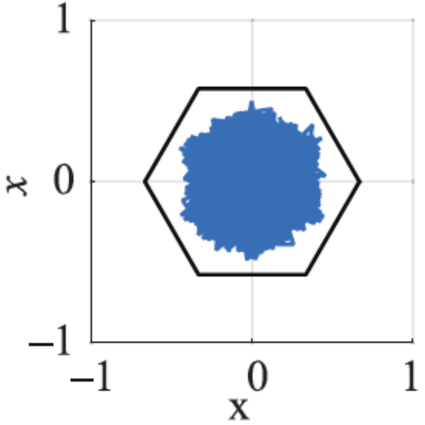
CCS-MPC

- Convergences to origin
- Noise introduces (minor) variations



FCS-MPC

- Candidate Control Lyapunov Function (CLF)
  provides upper bounds on flux, i.e. current, ripple
- In practice, FCS-MPC tends to do better than predicted
  if error is (heavily) penalized
  especially at low speed



Steady-State Behavior – Control Invariant Sets

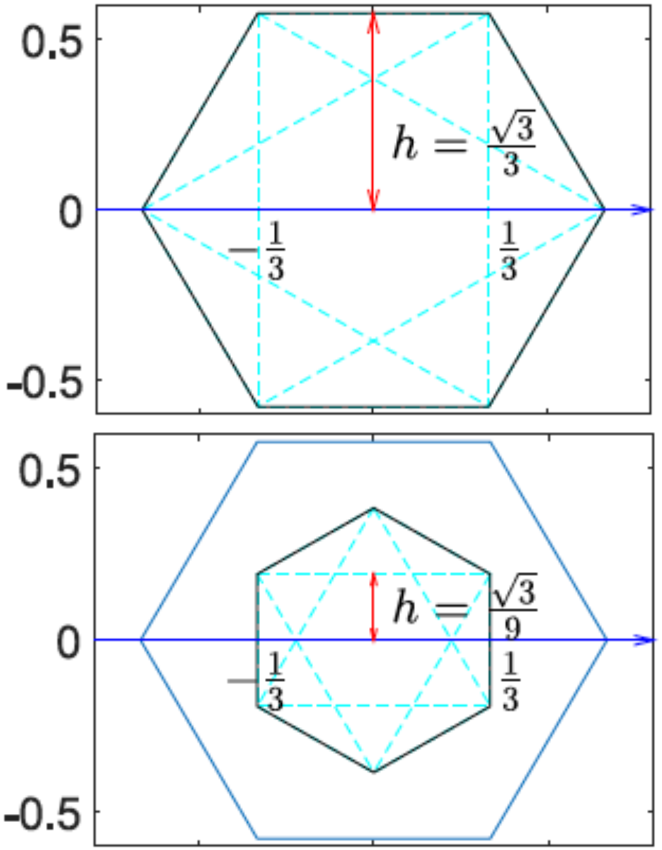
Definition of "steady-state" for FCS-MPC:

$$x \in V_\mathrm{d}$$

V_d can be constructed by 3 rectangles

- Parametrized with height h
- Upper bound $h = \frac{\sqrt{3}}{3}$ defines V_d
- Lower bound $h = \frac{\sqrt{3}}{9}$ closely resembles low speed control error

Rotated hexagon is **not** control invariant for $x \notin V_\mathrm{d}$

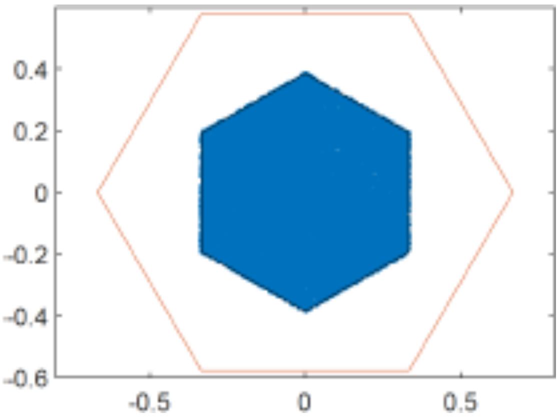


Steady-State Behavior – Definitions

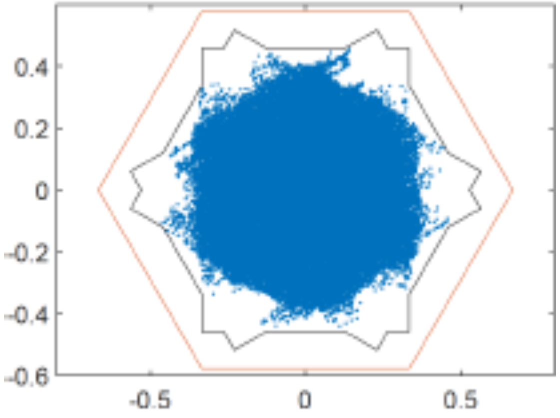
All subsets defined by $\frac{\sqrt{3}}{9} \leq h \leq \frac{\sqrt{3}}{3}$ are control invariant

iff $\overline{u} \in B\left(\frac{h}{3} + \frac{\sqrt{3}}{6}\right) \quad \Leftrightarrow$ reduced back-EMF

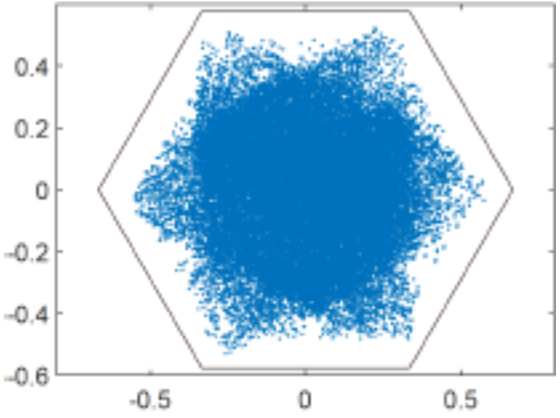
$h \leq \frac{\sqrt{3}}{9}$, ω = 50 rad/s



$h \leq \frac{38\sqrt{3}}{144}$, ω = 517 rad/s



$h \leq \frac{\sqrt{3}}{3}$, ω = 577 rad/s

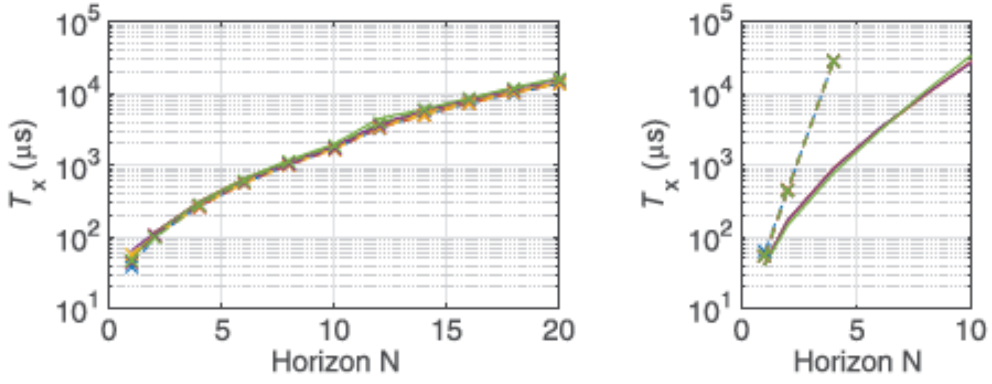


Steady-State Behavior – Computation Complexity

CCS: efficient solvers, e.g. fast gradient with warm start and early termination

FCS:

- Exploit Lyapunov constraint (V_{d}): ignore sequences that violate constraint
- Branch-and-bound (BnB): ignore if sequence exceeds best total cost



(a) CCS-MPC (b) FCS-MPC

Table 6.1.: Number of cost function evaluations

Horizon N	Full Enum. (8^N evaluations)	Opt. Enum. MEAN	Opt. Enum. MAX
1	8	1.4	4
2	64	3.9	12
3	512	7.1	40
4	4096	11.2	113
5	32768	16.9	261
6	262144	25.0	666
7	2097152	36.7	710
8	16777216	53.2	762

Optimization-based Observers

Position Sensorless

Y. Sun et. al. „ Unified Wide Speed Range IPM Sensorless Scheme Using Nonlinear Optimization,“ TPEL, 2017

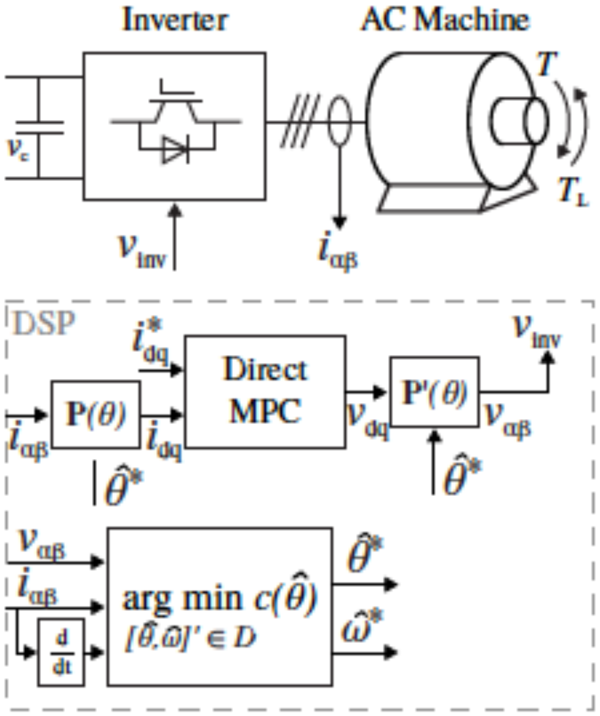
Optimization-based Observer – Approach

MPC-Approach

- Define control problem as cost function and constraints
- Rely on optimization tool to deliver expected outcome

Optimization-based observer

- Position/speed estimation as optimization problem
- Single block diagram for low and high speed
- Remove demodulation and filters
- Inherent support of CCS and FCS MPC



Optimization-based Observer – Definitions

PMSM dynamic model written in αβ

$$\bar{v}_{\alpha\beta} = \left(L_\Sigma \mathbf{I} + L_\Delta \bar{\mathbf{P}}(2\theta)\right) \dot{i}_{\alpha\beta} + 2L_\Delta \omega \mathbf{J} \bar{\mathbf{P}}(2\theta) i_{\alpha\beta} + \omega\psi q(\theta)$$

Implicit function

$$h(\hat{z}) = \left(L_\Sigma \mathbf{I} + L_\Delta \bar{\mathbf{P}}(2\hat{\theta})\right) \dot{i}_{\alpha\beta} + 2L_\Delta \hat{\omega} \mathbf{J} \bar{\mathbf{P}}(2\hat{\theta}) i_{\alpha\beta} + \psi\hat{\omega} q(\hat{\theta}) - \bar{v}_{\alpha\beta}$$

with estimates $\hat{z} = \left[\hat{\theta}, \widehat{\omega}\right]'$ and estimation error $\tilde{z} = [\tilde{\theta}, \widetilde{\omega}]'$

Instantaneous and independent estimation of position and speed

$$\tilde{z}^\star = \arg\min_{\tilde{z} \in \bar{\mathcal{D}}} \bar{c}(\tilde{z}) = \|\bar{h}(\tilde{z})\|^2$$

where $\bar{h}(\tilde{z}) = h(z - \tilde{z})$ to simplify the analysis.

Optimization-based Observer – Convexity

Theorem: convergence

Let $\bar{c}(0)$ be a **strict minimum** on the optimization domain $\overline{D}$, then $\tilde{z}^* = 0$

Corollary: strictly convex cost function

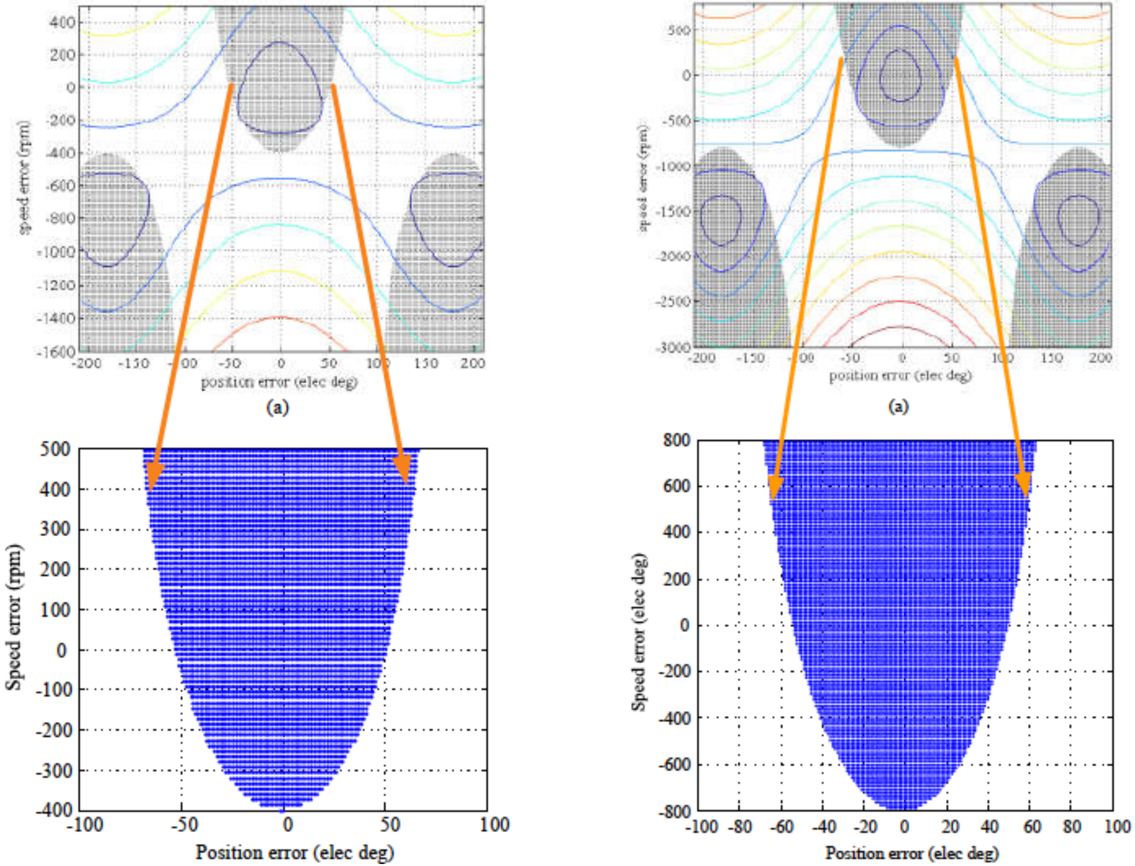
Let $\bar{c}(\tilde{z})$ be **strictly (pseudo) convex** on $\overline{D}$, then $\tilde{z}^* = 0$

Strict convexity depends on

- Parameters (machine type)
- Currents $i_{\alpha\beta}$ and perturbation $\dot{i}_{\alpha\beta}$

Optimization-based Observer – Examples: Convex Regions (gray)

IPMSM 400rpm

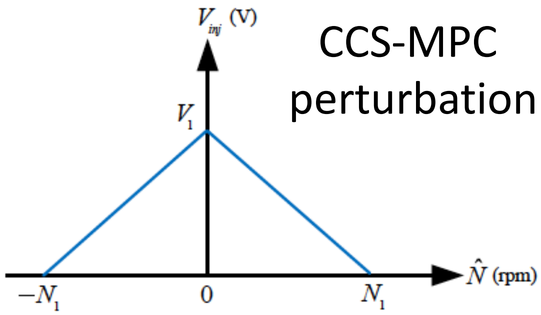


IPMSM 800rpm

Optimization-based Observer – Optimization Domain

Origin

- Simple criterion exists for strict convexity
  → Low speed requires perturbation



CCS-MPC perturbation

Optimization Domain

- Required to be a **convex** area where $\bar{c}(\tilde{z})$ is **strictly convex**
- Identifies an accurate lower bound for the **domain of convergence** for any position and speed sensorless

Optimization-based Observer – Example: Performance

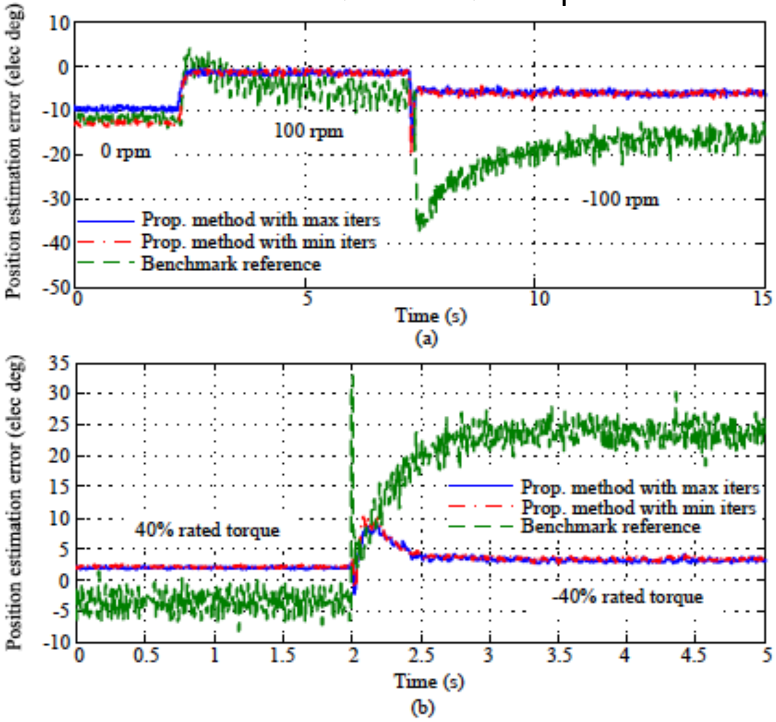
Comparison with traditional methods

- Similar computation complexity (few Newton steps required)
- Improved settling time by factor 40
- Similar parameter dependence

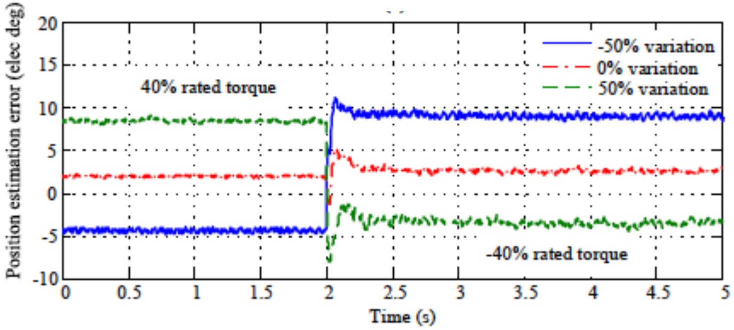
Position error in transient operation:

Top: 100rpm → -100rpm

Bottom: 40% → -40% torque



Position error with parameter mismatch



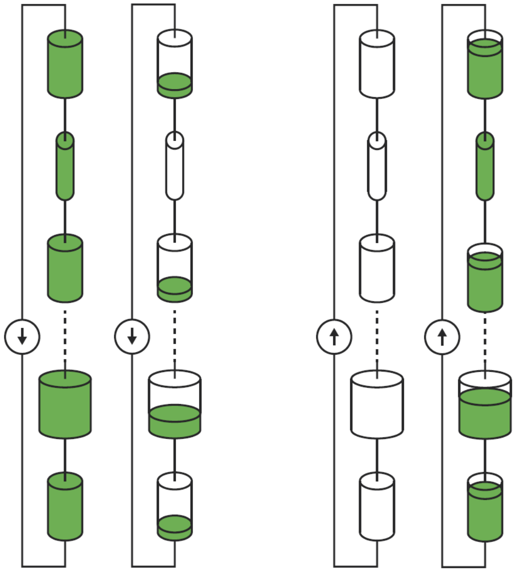
Novel Topologies

State-of-Charge Balancing

M. Preindl „A Battery Balancing Auxiliary Power Module with Predictive Control for Electrified Transportation,“ TIE, 2017

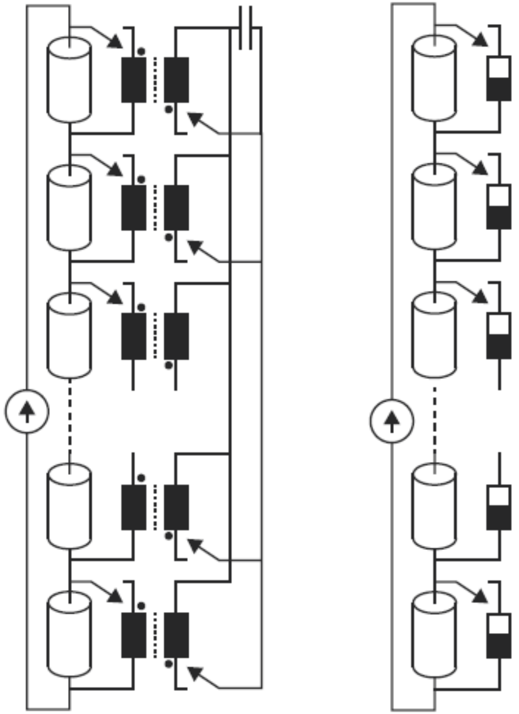
Battery Management: Charge Equalization Problem

- Battery stacks
  - Battery cells have varying parameters (capacity, etc.) → Balancing problem
  - Unbalanced strings
    - Low effective capacity
    - Exponential lifetime reduction with string length
  - Require balancing power electronics and control



Battery Management: Redistributive Topologies

- High-performance redistributive topologies
  e.g. capacitive exchange element
- Require active balancing links:
  isolated DC/DC converters
- Typically considered too expensive for EV
  → use dissipative topologies



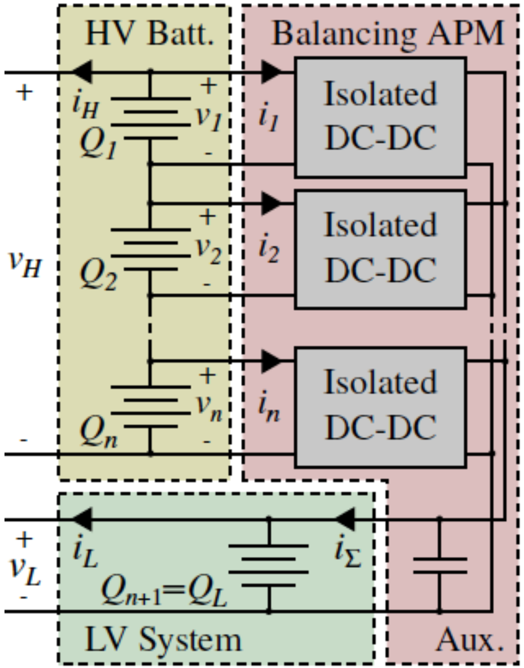
(e) Capacitive (a) Dissip.

Battery Management: BB-APM

Add functionality to balancing hardware

- Integrate auxiliary power module (APM)
  → supply auxiliary battery
- Replace dedicated auxiliary power module (APM)

Pack has (many) high-voltage cells
and one isolated low-voltage cell



(a) With LV battery

Battery Management: Charge Equalization Problem

- Battery stack model

$$\dot{x}(t) = \mathbf{B}u(t)$$

where $\mathbf{B} = \mathbf{Q}^{-1}\mathbf{TN}$ with topology matrix $\mathbf{T} \in \mathbb{R}^{n \times m}$

- State constraint $x \in [0,1]^n$
- Input constraint $u \in \mathcal{U} = \{u \in \mathbb{R}^m | \mathbf{H}u \leq K\}$
- Balancing problem:

Find $u(t) \in \mathcal{U}$ and time $\tau \in \mathbb{R}_+$ s.t.

$$\bar{x}(\tau) = \mathbf{L}x(\tau) = \mathbf{L}x(0) + \mathbf{LB}\int_0^\tau u(t)dt = \mathbf{0}$$

where $\mathbf{L} = \mathbf{I} - \frac{1}{n}\mathbf{1}\mathbf{1}'$

Battery Management: BB-APM Control

- BB-APM: two control goals
  - Balance high voltage cells
  - Charge low voltage cell
- MPC formulation

$$\min_{u[k]\in\mathcal{U}} \left\|(q_b\bar{\mathbf{R}} + q_c\mathbf{R})\tilde{x}[k+1]\right\|_q + \|r_l u[k]\|_q$$
$$\text{subject to } \tilde{x}[k+1] = \mathbf{L}x[k+1] - r;$$
$$x[k+1] = x[k] + \mathbf{B}u[k] + \mathbf{E}w[k] \in \mathcal{X}.$$

  - With reference $r = [0, \dots, 0,1]'$, known disturbance w
  - Cost with q-norm and weighting factors: q_b, q_c, and r_l.

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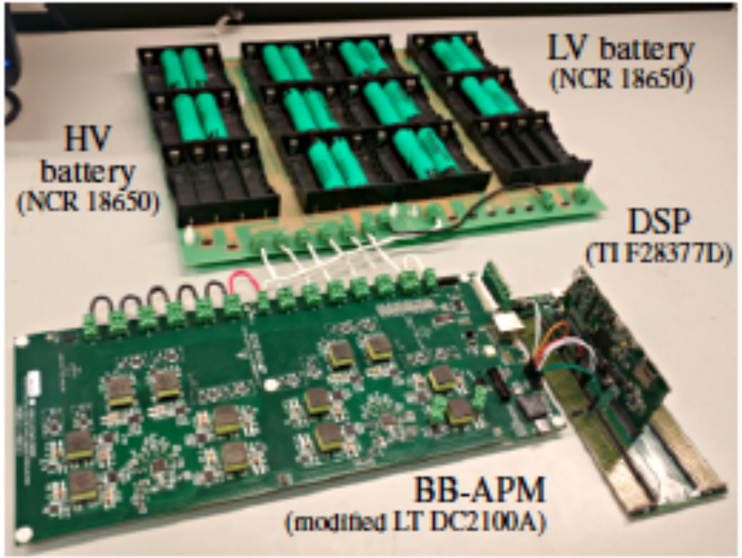
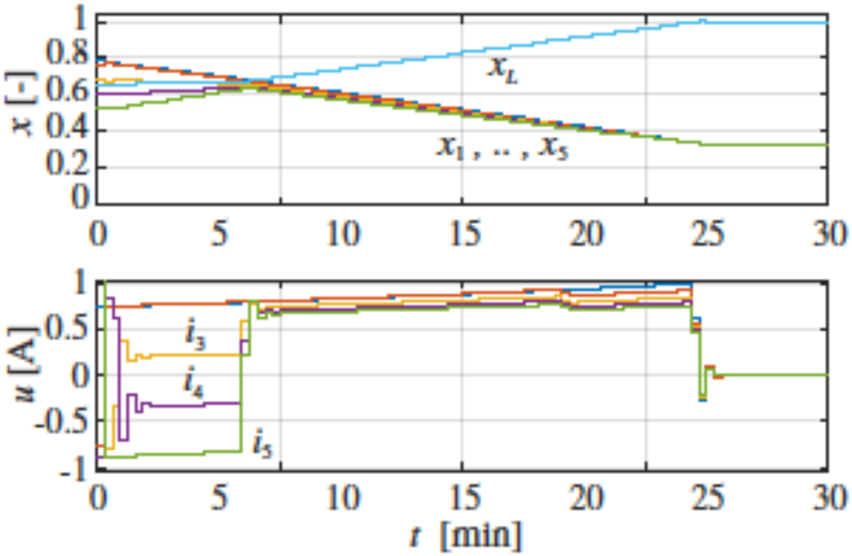
Battery Management: Example



(a) Experimental test bench



(c) Experimental results

Thank you.