Optimization-based Control and Estimation
PRECEDE 2017
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Ivy League Research University
Main Campus located in Manhattan, Morningside Heights

School of Engineering and Applied Science
~ 140 faculty members
~ 2,000 graduate students
~ 1,500 undergraduate students

Department of Electrical Engineering
• 36 faculty members
• Only one working in power
Motor Drives and Power Electronics Laboratory (MPLab)

- Founded 2016
- Located on Columbia main campus

People

- Laboratory members
  - 4 PhD, 1 MSc students
- Co-supervision
  - 3 PhD students
  - 1 post-doc, 1 research associate
- Project-based members

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Research
• Advanced control
  • Optimal control
  • Nonlinear control
  • Observers

• Power electronics
  • Wide-bandgap
  • High-frequency (MHz)

Applications
• Electric Vehicle Drivetrains
  • Traction drive systems
  • Energy storage systems
  • Power converters

Graphic: tesla.com
Optimization-based Control and Estimation
Introduction
Introduction

MPC Benefits
- Nonlinear Systems
- Multi-input multi-output
- Constraint handling

MPC Challenges
- “Classical” MPC stability theorem requires specific cost function, prediction horizon, and terminal constraint
- Convexity and computation efficiency
- Model accuracy
Virtual-flux model \((\lambda_{dq}, \lambda_{\alpha\beta})\)

- Any multiphase AFE and sinusoidal machines: IPMSM, SPMSM, RSM, IM
- No parameters in dynamic model

MPC schemes

- FCS preferable at low \(f_s\) \(\rightarrow\) needs to detect ripple
- CCS preferable at high \(f_s\) \(\rightarrow\) ripple is handed off to modulator

Tendency to use unconventional cost functions

- Incompatibility with MPC theory: stability and robustness
Summary

• Single block is not be best for everything

• Stability and robustness concepts for any cost function

• MPC concepts are applicable to estimation

• MPC enables new power electronic topologies
Model Predictive Torque Control using Virtual Fluxes Concept
Virtual flux $\lambda$ space
- Compute $\lambda$ with static (nonlinear) map
- Linear dynamic model without parameters
- Prediction is parameter independent

Setpoint calculation
- Separate dynamic control from choosing setpoints
- Nonlinear map: torque $\rightarrow$ current (or flux)

Regulation problem: $x \rightarrow 0$
- Tracking $\rightarrow$ regulation problem
- Combine feedback and feedforward control
Model Predictive Torque Control using Virtual Fluxes
Constrained MTPA

M. Preindl, S. Bolognani „Optimal State Ref. Computation with Constrained MTPA Criterion for PM Drives,“ TPEL, 2015
Model

Stator dynamics

\[
\dot{\lambda}_{dq}(t) = -\omega J \lambda_{dq}(t) + \bar{\nu}_{dq}(t)
\]

Current-flux map

\[
\lambda_{dq}(t) = l \circ i_{dq}(t) \approx L i_{dq}(t) + \psi_{dq}
\]

Torque equation

\[
T = \frac{3}{2} p \ i_{dq}' J \lambda_{dq} = \frac{3}{2} p \ (\psi + (L_d - L_q) i_d) i_q
\]

\[
J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}; \quad L = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix}; \quad \psi_{dq} = \begin{bmatrix} \psi \end{bmatrix};
\]
Constraints

Current constraint

\[ i_{dq} \in J \overset{\text{def}}{=} \{ i_{dq} \in \mathbb{R}^2 \mid \|i_{dq}\| \leq I_r \} \]

Voltage constraint

\[ \bar{v}_{dq} \in \bar{V} \overset{\text{def}}{=} \{ \bar{v}_{dq} \in \mathbb{R}^2 \mid \|\bar{v}_{dq}\| \leq \bar{v}_r \overset{\text{def}}{=} \rho_v v_r \} \]

Flux constraint

\[ \lambda_{dq} \in \Lambda \overset{\text{def}}{=} \{ \lambda_{dq} \in \mathbb{R}^2 \mid |\omega|\|\lambda_{dq}\| \leq \bar{v}_r \} \]

Speed constraint

\[ \frac{|\omega|}{\bar{v}_r} \leq \chi_m \overset{\text{def}}{=} \begin{cases} \frac{1}{|\psi - L_d I_r|} & \text{if } \frac{\psi}{L_d} > I_r, \\ \infty & \text{otherwise.} \end{cases} \]
Constrained MTPA

Definition: optimal states

Let $i^*_d, \lambda^*_d$ be optimal if they produce $T$ according to

\[
\begin{align*}
\text{minimize} & \quad ||i_d|| \\
\text{subject to} & \quad ||i_d|| \leq I_r; \\
& \quad |\omega||\lambda_d| \leq \nu_r; \\
& \quad \lambda_d = L_i d_q + \psi d_q; \\
& \quad 3/2p' \ i_d' J \lambda_d = T; \\
& \quad |T| \leq T_m(\omega, \nu_r)
\end{align*}
\]

Nonconvex (NP-hard in general), possibly infeasible $\Rightarrow$ split into subproblems
Constrained MTPA – Example: Base Mode

(a) Current space

(b) Current magnitude vs. torque
Constrained MTPA – Example: Constant Power Mode
Constrained MTPA – Example: Reduced Power Mode

(a) Current space
(b) Current magnitude vs. torque
Constrained MTPA – $T_m$ and $T_i$ definitions

**Definition: Maximum Torque**

$$T_m \overset{\text{def}}{=} \frac{3}{2} p \max_{i_{dq}, \lambda_{dq}} i_{dq}' J \lambda_{dq}$$

subject to $\|i_{dq}\| \leq I_r$; 
$|\omega|\|\lambda_{dq}\| \leq \bar{\nu}_r$; 
$\lambda_{dq} = L i_{dq} + \psi_{dq}$

**Definition: Intersection Torque**

$$T_i \overset{\text{def}}{=} \frac{3}{2} p \max_{i_{dq}, \lambda_{dq}} i_{dq}' J \lambda_{dq}$$

subject to $\|i_{dq}\| \leq I_r$; 
$|\omega|\|\lambda_{dq}\| \leq \bar{\nu}_r$; 
$\lambda_{dq} = L i_{dq} + \psi_{dq}$; 
$i_{dq} \in \text{MTPA}$

- Problems are feasible;
- Still non-convex but can be solved efficiently due to low dimension
Constrained MTPA – $T_m$ and $T_i$ with infinite max. speed
Constrained MTPA – $T_m$ and $T_i$ with finite max. speed
Constrained MTPA – Optimal Reference Computation

Find maximum torque $T_m$

Find intersection torque $T_i$

Find optimal states
- Locate trajectory ($\text{MTPA, } \partial\Lambda$) with $T_m, T_i$
- Compute $i_{dq,\text{ref}}$ (or $\lambda_{dq,\text{ref}}$)
Constrained MTPA – Example: Low and High Speed Operation

Find maximum torque

Transient operation (speed step) standstill to field-weakening

CCS-MPC operation points

FCS-MPC operation points
Rated $L$ and $\psi_{dq}$ are typically suboptimal.
Constrained MTPA – Model Refinement

Approach

- Optimize model **locally** (area enclosed by MTPA, $\partial \Lambda$, and $\partial I$)
- Using well known operating points (rated operation point, short circ. current)

**Parameters** $M \rho = K$

\[
\begin{bmatrix}
\frac{3}{2} i_{qr} & \frac{3}{2} i_{dr} i_{qr} & \frac{-3}{2} i_{dr} i_{qr} & -T_r \\
1 & i_{dr} & i_{qr} - i_{dr} & i_{qr} \\
1 & i_{dc} & & \\
\end{bmatrix}
\begin{bmatrix}
\psi \\
L_d \\
L_q \\
1/p \\
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
\lambda_{dr} \\
\lambda_{qr} \\
0 \\
\end{bmatrix}
\]

- Least square solution: $\rho = M^+ K \to$ not a general model, e.g. $p \notin \mathbb{N}$
Constrained MTPA – Example: Model Refinement

Model (solid) and measured (dashed) characteristics
Model Predictive Torque Control using Virtual Fluxes
Transient Behavior

Transient Behavior – Dynamic model

Stator **dynamic equation**

\[ \tilde{\lambda}_{\alpha\beta}^+ = \tilde{\lambda}_{\alpha\beta} + \bar{v}_{\alpha\beta} \]

Where \( \tilde{\lambda}_{\alpha\beta} = \Lambda^{-1}_r \lambda_{\alpha\beta} \) and \( \Lambda_r = T_s v_c \)

Transform tracking into **regulation problem**

\[ x^+ = x + u \]

with control error \( x = \tilde{\lambda}_{\alpha\beta} - \tilde{r}_{\alpha\beta} \) and input

The terminal voltage \( \bar{v}_{\alpha\beta} \)

- Feedback controller \( u : MPC \)
- Feedforward controller \( \bar{u} : \text{adjustment for rotation} \)

\[ \bar{u} \approx -(I - T^{-1}_{dq}(T_s \omega))\tilde{r}_{\alpha\beta} \]
Transient Behavior – Terminal Voltage and Input constraints

CCS and FCS input constraints (Note: $U_s \overset{\text{def}}{=} V_s - \bar{u}$ does not contain origin)

(a) Set $V_s$ and $V_d = \text{hull } V_s$

(b) Set $U_s$ and $U_d = \text{hull } U_s$
Transient Behavior – FCS Candidate Lyapunov Function

\[
\Gamma(x) = \min_{x \in \Omega} X_1
\]

\[
\Omega = \{ u \in \mathbb{R}^2 \mid Hx \leq X_1 \}
\]
Transient Behavior – Preset and Robust Control Invariance

Preset applied to sublevel set $\Omega_{\gamma}$

The preset $O(\Omega_{\gamma})$ is the set of all states $x \in \mathbb{R}^2$ that can be driven to the $\Omega_{\gamma}$ by an admissible control input $u \in U$

$$O_B(\Omega_{\gamma}) = \{ x \in \mathbb{R}^2 | \exists u \in U : x + u \in \Omega_{\gamma} \}$$

Robust control invariance

The set $\Omega_{\gamma}$ is said to be robust control invariant iff $\Omega_{\gamma} \subseteq O(\Omega_{\gamma}) - B$, where $B$ is an arbitrarily small ball with radius $b$

Iff $\Omega_{\gamma}$ is robust control invariant $\exists u \in U$ s.t. $\Gamma(x^+) - \Gamma(x) < -b$. 
Using FCS, the sublevel $\Omega_\gamma$ is (robust) control invariant if large enough
Transient Behavior – Example: presets and control invariance

(c) Transients \( x \notin V_d \): \( O(\Omega(\gamma)) \ominus B \) contains \( \Omega(\gamma) \)

(d) Steady state \( x \in V_d \): \( O(V_d) \ominus B \) is rotating and contains \( V_d \)
**Theorem:** global and robust set stabilizability

Let $\bar{u} \in V_d - B$, then $\exists u \in U_s$ s.t.

$$\Gamma(x^+) - \max\left(\Gamma(x), \frac{1}{\sqrt{3}} + b\right) < -b$$

The Lyapunov function can be decreased every time step by $-b$ until the level $\frac{1}{\sqrt{3}}$

**Corollary:** set convergence

There exists a sequence $u_0, u_1, ..., u_k, ... \in U_s$ s.t.

$$\lim_{k \to \infty} x_k \in V_d$$

The control error converges to $V_d$
The CCS system inherits the FCS properties (without lower bound)

**Theorem:** global and robust stabilizability

Let \( \bar{u} \in V_d - B \), then \( \exists u \in U_d \) s.t.

\[
\Gamma(x^+) - \max(\Gamma(x), b) < -b
\]

The Lyapunov function can be decreased every time step by \(-b\) until the level \( \frac{1}{\sqrt{3}} \)

**Corollary:** convergence to origin

There exists a sequence \( u_0, u_1, \ldots, u_k, \ldots \in U_s \) s.t.

\[
\lim_{k \to \infty} x_k \in 0
\]

The control error converges to origin
Transient Behavior – Robust Lyapunov-based MPC

Constraint Finite Time Optimal Control (CFTOC)

- Enforce stability with contraction constraint
  CCS: any norm; FCS: specific candidate CLF
- Holds for any cost function
- Simplification possible for horizon N=1

\[
\begin{align*}
    & \text{minimize } J(\cdot) \\
    & \text{subject to } x_{j+1} = x_j + u_j \\
    & u_j \in U_j \overset{\text{def}}{=} \mathcal{V} - \bar{u}_j \\
    & \Gamma(x_0 + u_0) - \max (\Gamma(x_0), \tilde{\gamma} + b) \leq -b \\
    & \tilde{\gamma} = \frac{1}{\sqrt{3}} \text{ for FCS; } \tilde{\gamma} = 0 \text{ for CCS}
\end{align*}
\]
Transient Behavior – Example: Parameter Robustness
Transient Behavior – Example: Parameter Robustness

Smaller L by factor 100

Rated parameters

Larger L by factor 100
MPC for PM Synchronous Motor Drives
Steady-State Behavior

X. Yong, M. Preindl „Smallest Control Invariant Set and Error Boundaries of FCS-MPC for PMSM ,“ APEC, 2017
Steady-State Behavior – Observations

CCS-MPC
- Convergences to origin
- Noise introduces (minor) variations

FCS-MPC
- Candidate Control Lyapunov Function (CLF) provides upper bounds on flux, i.e. current, ripple
- In practice, FCS-MPC tends to do better than predicted if error is (heavily) penalized especially at low speed
Steady-State Behavior – Control Invariant Sets

Definition of “steady-state” for FCS-MPC:

\( x \in V_d \)

\( V_d \) can be constructed by 3 rectangles

- Parametrized with height \( h \)
- Upper bound \( h = \frac{\sqrt{3}}{3} \) defines \( V_d \)
- Lower bound \( h = \frac{\sqrt{3}}{9} \) closely resembles low speed control error

Rotated hexagon is not control invariant for \( x \notin V_d \)
Steady-State Behavior – Definitions

All subsets defined by \( \frac{\sqrt{3}}{9} \leq h \leq \frac{\sqrt{3}}{3} \) are control invariant iff
\[
\bar{u} \in B\left(\frac{h}{3} + \frac{\sqrt{3}}{6}\right) \iff \text{reduced back-EMF}
\]

- \( h \leq \frac{\sqrt{3}}{9}, \ \omega = 50 \text{ rad/s} \)
- \( h \leq \frac{38\sqrt{3}}{144}, \ \omega = 517 \text{ rad/s} \)
- \( h \leq \frac{\sqrt{3}}{3}, \ \omega = 577 \text{ rad/s} \)
Steady-State Behavior – Computation Complexity

**CCS:** efficient solvers, e.g. fast gradient with warm start and early termination

**FCS:**
- Exploit Lyapunov constraint ($V_d$): ignore sequences that violate constraint
- Branch-and-bound (BnB): ignore if sequence exceeds best total cost

<table>
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<th>Horizon N</th>
<th>Full Enum. ($8^N$ evaluations)</th>
<th>Opt. Enum. MEAN</th>
<th>MAX</th>
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Optimization-based Observers
Position Sensorless

Optimization-based Observer – Approach

**MPC-Approach**

- Define control problem as cost function and constraints
- Rely on optimization tool to deliver expected outcome

**Optimization-based observer**

- Position/speed estimation as optimization problem
- Single block diagram for low and high speed
- Remove demodulation and filters
- Inherent support of CCS and FCS MPC
Optimization-based Observer – Definitions

PMSM dynamic model written in αβ

\[ \bar{v}_{\alpha\beta} = (L\Sigma I + L_\Delta \bar{P}(2\theta)) \dot{i}_{\alpha\beta} + 2L_\Delta \omega J \bar{P}(2\theta) i_{\alpha\beta} + \omega \psi q(\theta) \]

Implicit function

\[ h(\hat{z}) = \left( L\Sigma I + L_\Delta \bar{P}(2\hat{\theta}) \right) \dot{i}_{\alpha\beta} + 2L_\Delta \hat{\omega} J \bar{P}(2\hat{\theta}) i_{\alpha\beta} + \hat{\omega} q(\hat{\theta}) - \bar{v}_{\alpha\beta} \]

with estimates \( \hat{z} = [\hat{\theta}, \hat{\omega}]' \) and estimation error \( \tilde{z} = [\tilde{\theta}, \tilde{\omega}]' \)

Instantaneous and independent estimation of position and speed

\[ \tilde{z}^* = \arg \min_{\tilde{z} \in \bar{D}} \bar{c}(\tilde{z}) = \|\bar{h}(\tilde{z})\|^2 \]

where \( \bar{h}(\tilde{z}) = h(z - \tilde{z}) \) to simplify the analysis.
Theorem: convergence
Let \( \bar{c}(0) \) be a strict minimum on the optimization domain \( \bar{D} \), then \( \tilde{z}^* = 0 \)

Corollary: strictly convex cost function
Let \( \bar{c}(\tilde{z}) \) be strictly (pseudo) convex on \( \bar{D} \), then \( \tilde{z}^* = 0 \)

Strict convexity depends on
• Parameters (machine type)
• Currents \( i_{\alpha\beta} \) and perturbation \( i_{\alpha\beta} \)
Optimization-based Observer – Examples: Convex Regions (gray)

IPMSM 400rpm

IPMSM 800rpm
Optimization-based Observer – Optimization Domain

Origin

• Simple criterion exists for strict convexity
  → Low speed requires perturbation

Optimization Domain

• Required to be a convex area where $\bar{c}(\hat{Z})$ is strictly convex
• Identifies an accurate lower bound for the domain of convergence for any position and speed sensorless

CCS-MPC perturbation
Comparison with traditional methods

- Similar computation complexity (few Newton steps required)
- Improved settling time by factor 40
- Similar parameter dependence

Position error in transient operation:
- Top: 100rpm → -100rpm
- Bottom: 40% → -40% torque

Position error with parameter mismatch:
- 40% rated torque
- -40% rated torque
- 0% variation
- 50% variation

Charts show position error over time with different conditions.
Novel Topologies
State-of-Charge Balancing

M. Preindl „A Battery Balancing Auxiliary Power Module with Predictive Control for Electrified Transportation,“ TIE, 2017
Battery Management: Charge Equalization Problem

- Battery stacks
  - Battery cells have varying parameters (capacity, etc.)
    → Balancing problem
  - Unbalanced strings
    - Low effective capacity
    - Exponential lifetime reduction with string length
  - Require balancing power electronics and control
Battery Management: Redistributive Topologies

- High-performance redistributive topologies e.g. capacitive exchange element
- Require active balancing links: isolated DC/DC converters
- Typically considered too expensive for EV → use dissipative topologies
Add functionality to balancing hardware

- Integrate auxiliary power module (APM) → supply auxiliary battery
- Replace dedicated auxiliary power module (APM)

Pack has (many) high-voltage cells and one isolated low-voltage cell
Battery Management: Charge Equalization Problem

- Battery stack model
  \[ \dot{x}(t) = Bu(t) \]
  where \( B = Q^{-1}TN \) with topology matrix \( T \in \mathbb{R}^{n \times m} \)
- State constraint \( x \in [0,1]^n \)
- Input constraint \( u \in \mathcal{U} = \{u \in \mathbb{R}^m | Hu \leq K\} \)
- Balancing problem:
  Find \( u(t) \in \mathcal{U} \) and time \( \tau \in \mathbb{R}_+ \) s.t.
  \[ \bar{x}(\tau) = Lx(\tau) = Lx(0) + LB \int_0^\tau u(t) dt = 0 \]
  where \( L = I - \frac{1}{n}11' \)
Battery Management: BB-APM Control

- BB-APM: two control goals
  - Balance high voltage cells
  - Charge low voltage cell
- MPC formulation

\[
\min_{u[k] \in \mathcal{U}} \left\| (q_b \mathbf{R} + q_c \mathbf{R}) \tilde{x}[k + 1] \right\|_q + \| r_i u[k] \|_q
\]

subject to

\[
\tilde{x}[k + 1] = L x[k + 1] - r;
\]

\[
x[k + 1] = x[k] + B u[k] + E w[k] \in \mathcal{X}.
\]

- With reference \( r = [0, \ldots, 0, 1]' \), known disturbance \( w \)
- Cost with q-norm and weighting factors: \( q_b, q_c, \) and \( r_i \).
Battery Management: Example

(a) Experimental test bench

(c) Experimental results
Thank you.